

Natural Convective Behavior and Entropy Generation of a Reacting Nanofluid in a Partially Porous Dome-shaped Cavity with a Fin

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Abstract: This study investigates natural convection, thermal energy transfer, and entropy generation behaviors of ethanol–air mixture containing Al_2O_3 nanoparticles in a partially porous dome-shaped cavity. The problem is solved using the Galerkin finite element method (GFEM). Results provide that reducing Darcy number (Da) restricts fluid flow and enhances conduction effects in the system. Increasing nanoparticles' volume fraction (ϕ) improves heat transfer and lowers peak temperature ($\theta_{\max}$) by 19%, although the increase in viscosity decreases the maximum stream function ($\psi_{\max}$) by 7%. In contrast, stronger exothermic reactions enhance thermal buoyancy, increasing circulation strength by up to 34% while accelerating species consumption. Entropy generation analysis shows that increasing L_P and ϕ reduces global entropy generation (S_{global}) by 26%–46%, leading to lower thermodynamic losses. The Bejan number (Be) decreases to nearly 55%–60% at high Ra, indicating a shift toward friction dominated irreversibility, while the porous structure strongly controls the entropy generation ratio (EGR).

Keywords: Natural convection; Entropy generation; Reacting fluid; Dome-shaped cavity; Partially porous cavity.

1. INTRODUCTION

The transport of thermal energy and chemical species driven exclusively by buoyancy forces is a key component of thermal engineering systems and applications. This complex physical process, widely recognized as double diffusive natural convection, occurs when fluid density variations are simultaneously induced by temperature gradients and concentration gradients. A brief review of the literature shows that, due to its practical importance, the study of double diffusive natural and mixed convection heat transfer in enclosures has been extensively studied over the past few decades. Convective heat transfer and fluid flow analysis in a partially heated trapezoidal enclosure was studied by Hamid *et al.* [1], while Teamah *et al.* [2] investigated steady, laminar, double diffusive mixed convection in a rectangular lid driven cavity under the combined buoyancy effects of thermal and mass diffusion. Sezai and Mohamad [3] investigated three dimensional double diffusive convection in a cubic enclosure. Their results revealed complex secondary flow structures on transverse planes beyond the predictive capability of traditional two dimensional models. More recently, Sheremet *et al.* [4] expanded on this by detailing how localized boundary conditions influence convective species transport. The industrial reliance on

this phenomenon is massive. Systems ranging from high power electronic cooling to solar energy harvesting, oil refining, geothermal extraction, and food processing are strongly influenced by natural convection heat transfer [5, 6]. Natural convection is attractive for commercial and industrial applications because of its passive nature. It operates without external mechanical devices. Consequently, it reduces the additional energy consumption required by active cooling systems [7]. However, modern engineering systems are becoming smaller and more thermally demanding. They require models that account for complex geometries, composite fluid domains, and coupled heat and species transport [8, 9].

For several decades, computational fluid dynamics studies mainly focused on square or rectangular cavities to analyze convective fluid flow. These simple geometries were useful for developing numerical models. However, they cannot accurately capture fluid structure interactions in practical engineering and architectural systems. Motivated by real world applications, recent thermofluid studies have shifted toward irregular geometries, including trapezoidal, parallelogram, and wavy wall enclosures [10, 11, 12]. Among these irregular geometries, the dome-shaped cavity has attracted considerable research interest.

They are widely used in green building architectures, advanced nuclear reactor containment vessels, and large scale solar central receiver systems. These

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applications require uniform stress distribution and effective heat retention. From a fluid dynamics point of view, the continuous curvature of a dome significantly modifies boundary layer behavior. Rising thermal plumes are deflected by the sharp 90° corners of flat sided rectangular enclosures. In contrast, the smooth dome geometry allows buoyancy driven hot fluid to turn and recirculate more smoothly. Bairi [13] first examined the effect of inclination angle on heat transfer in hemispherical cavities. Subsequently, Yıldız *et al.* [14] demonstrated that specific dome geometries can enhance circulation, thereby improving heat transfer and reducing entropy generation. Extending this line of research, Javed and Saha [15] investigated conjugate natural convection in a dome-shaped partially porous cavity containing a solid cylinder. More recently, Nithiya and Prabhakar [16] studied natural convection behaviors in inclined dome enclosures. They analyzed thermodynamic optimization of the system using local entropy generation and Bejan number.

The flow field in such irregular enclosures is difficult to control, so advanced passive strategies are required. Natural convection heat transfer and fluid flow in partially porous cavities occur in a wide range of environmental and industrial applications. According to the specific application, the confined layers may be separated vertically or horizontally. Beckermann *et al.* [17, 18] performed a comprehensive analysis to study natural convection with fluid and porous layers, finding that fluid transport into the porous medium significantly alters the streamline patterns. An extensive analytic study of natural convection flow in a partially porous layer cavity was reported by Gobin *et al.* [19, 20], showing that coupling of flow transfer with combined buoyancy forces induces a nonmonotonic behavior with minimum in heat transfer. Turning to layered geometries, Siavashi *et al.* [21] investigated mixed convection heat transfer in a partially porous square cavity with a rotating cylinder. They identified optimal combinations of angular velocity and porous parameters for maximizing the average Nusselt number. Malashetty *et al.* [22] analyzed double diffusive convection in a fluid saturated porous layer under local thermal nonequilibrium. Their study showed that competition between thermal and solutal diffusion leads to oscillatory and subcritical instabilities. A confined porous layer is introduced adjacent to a clear fluid region. This configuration provides a dual advantage. The porous solid matrix offers a large specific surface area that enhances thermal dispersion, while the clear fluid region maintains high convective velocities. Accurate modeling of this macroscopic interface

requires advanced mathematical formulations such as the generalized Darcy–Brinkman–Forchheimer model. This model accounts for boundary friction and nonlinear inertial drag forces [23].

Even in an optimally designed partially porous cavity, conventional base fluids have low thermal conductivity. This remains a major limitation. This challenge led to the development of nanofluids, which are engineered by dispersing nanometer sized solid particles uniformly in a base fluid and have significantly improved heat transfer performance [24]. Chon *et al.* [25] considered empirical correlations have shown that Brownian motion is the main mechanism responsible for enhancing thermal conductivity, particularly with increasing temperature and decreasing particle size. Furthermore, Esfahani and Bordbar [26] studied conjugate heat transfer, establishing that high conductivity nanoparticles maximize heat transfer, while lower conductivity variants optimize species transport. Despite the extensive use of Al₂O₃ nanofluids in computational studies, a major limitation still persists in the literature, where most studies assume constant thermophysical properties for nanofluids. In reality, the dynamic viscosity and thermal conductivity of a nanofluid are intensely sensitive to localized temperature fluctuations [27]. Constant property assumption makes the results less accurate and affects predictions of flow and heat transfer [28]. Therefore, integrating temperature dependent, variable thermophysical properties for the Al₂O₃ nanofluid is an absolute physical necessity for yielding simulations with genuine predictive power.

In addition to changing the working fluid flow, using highly conductive extended surfaces is an effective way to improve heat transfer [29]. A T-shaped fin acts as a strong obstruction to the flow field. For instance, Kundu and Bhanja *et al.* [30] developed a nonlinear analytical model for a constructal T-shaped fin with variable thermal properties and used the Adomian decomposition method to optimize its geometry for maximum heat transfer at a fixed volume. As buoyant heated fluid rises along the fin, the transverse section of the fin can interrupt the thermal plume and redirect the flow laterally within the cavity, thereby modifying the local circulation pattern and thermal boundary layer development [31]. The physical complexity of the system deepens substantially when the confined working fluid is not chemically inert. In a wide array of industrial applications, including chemical vapor deposition (CVD) chambers, catalytic combustors, and advanced metallurgical processing units, chemical

reactions occur directly within the fluid phase [32]. The Arrhenius law is used to describe the chemical reaction. The reaction is exothermic. It generates heat within the domain and acts as a localized internal heat source. Localized heat generation or absorption changes the internal temperature field. It also alters the buoyancy forces that drive the convective flow. Mass diffusion of chemical species occurs simultaneously with heat transfer. As a result, the local fluid density depends on both the temperature and concentration fields [33].

Concerns over energy conservation and environmental sustainability are growing. This has increased interest in the optimization of thermal systems. System performance cannot be assessed by energy analysis alone. Thermodynamic irreversibilities must also be evaluated using the second law of thermodynamics. In this regard, entropy generation analysis has become a useful tool to identify sources of exergy loss and improve overall system efficiency. Bejan [34] focused on the different reasons behind entropy generation in applied thermal engineering, successfully mapping spatial irreversibility to optimize geometric parameters. In fundamental systems, Kumar *et al.* [35] applied the second law of thermodynamics to predict the nature of irreversibility in circular microscales, determining that frictional entropy generation vastly overshadows heat transfer irreversibility. Additionally, Wang *et al.* [36] analyzed entropy generation and energy losses in a hydrogen enriched methane air mixture in a micro planar combustor, showing that hydrogen addition reduces irreversibilities from heat conduction and mass diffusion while maintaining overall system performance.

This second law analysis is increasingly focused on heterogeneous and advanced fluid systems. Marzougui *et al.* [37] analyzed entropy generation in a trapezoidal porous cavity, determining that total thermodynamic irreversibility is highly sensitive to buoyancy forces. Lam and Prakash [38] investigated entropy generation in a fluid saturated porous cavity for natural convection with localized heat sources, while Siavashi *et al.* [39] analyzed the importance of thermal boundary conditions inside a porous enclosure utilizing entropy generation minimization. Recently, researchers have extended this analysis to nanofluids. The demand for sustainable energy systems is increasing. This has driven the growing interest in nanofluid based studies [40]. Rasool *et al.* [41] further investigated entropy generation in the laminar natural convection of a Williamson nanofluid. The Bejan number (Be) is widely used to analyze the different irreversibility mechanisms [42]. It represents the

ratio of entropy generated by heat and mass transfer to the total entropy generation.

A careful review of recent heat and species transport studies reveals a clear research gap. Over the past decades, many works have investigated natural convection, dome-shaped cavities, partially porous media, nanofluids with variable properties, and entropy generation. However, these phenomena have mostly been studied separately or in simplified forms. To the best of authors' knowledge, the coupled effects of heat transfer, fluid flow, species transport, and entropy generation in a T-shaped fin inside a dome-shaped partially porous cavity have not yet been reported. In addition, ethanol-based reacting Al_2O_3 nanofluids under non boussinesq conditions remain largely unexplored in heat and mass transfer literature.

Motivated by this gap, the present study numerically investigates the thermo fluid behavior of a chemically reacting Al_2O_3 nanofluid in a dome-shaped cavity. The configuration includes a partially porous layer to regulate flow resistance and a highly conductive T-shaped fin to enhance flow redistribution. Temperature dependent thermophysical properties are incorporated to ensure physical realism, and the Darcy-Brinkman-Forchheimer model is employed for the porous medium. In addition, a second law analysis is performed to evaluate local entropy generation and the Bejan number. Overall, this study provides a comprehensive understanding of the coupled thermofluid and thermodynamic characteristics of the system and its implications for thermal performance enhancement.

2. PROBLEM DESCRIPTION AND MATHEMATICAL MODEL

The current paper examines a detailed numerical analysis of the natural convective heat and species transport characteristics of a chemically reacting nanofluid (ethanol-based Al_2O_3) flow in a dome-shaped cavity that has three separate zones: a porous region in the middle and two clear nanofluid regions in the top and bottom. The analysis considers a single-step exothermic reaction, as defined in Eq. (1). A uniform initial distribution of oxygen from the surrounding air is assumed within the cavity. Internal heat generation from ethanol oxidation governs the system dynamics. As the reaction proceeds, the released thermal energy generates large localized temperature variations, which causes density variations that, under gravitational effects that induced natural convection due to buoyancy effects.

$$\text{Fuel} + \text{Oxidizer} \rightarrow \text{P} + \text{Heat}, \quad \text{Reactionrate} = A c_F c_g e^{-E/(RT)} \quad (1)$$

The physical domain D formulated for this investigation considers a stable, two-dimensional, incompressible laminar flow within a dome-shaped cavity of height L_y and width L_x featuring a partially porous region with a thickness $L_p = \frac{L_y}{3}$, which is configured horizontally at the mid height ($L_f = \frac{L_x}{2}$) of the cavity with a T-shaped fin, comprising a horizontal stem of length $L_{fin} = 0.18L_y$ and a vertical cap of width $W_{fin} = 0.18L_x$, is positioned centrally on the bottom wall; the fin possesses a constant thickness $t_{fin} = 0.02L_y$ throughout its structure. The side boundaries are defined by the parametric functions $x_L(s)$ and $x_R(s)$ for $s \in [0,1]$, expressed as:

$$\begin{aligned} x_L(s) &= -a \sin(\pi s) + 0.2s \\ x_R(s) &= a \sin(\pi s) + 0.8s + (1-s) \end{aligned}$$

where a denotes the curvature amplitude ($a = 0.13$). The schematic diagram of the problem is presented in Figure 1.

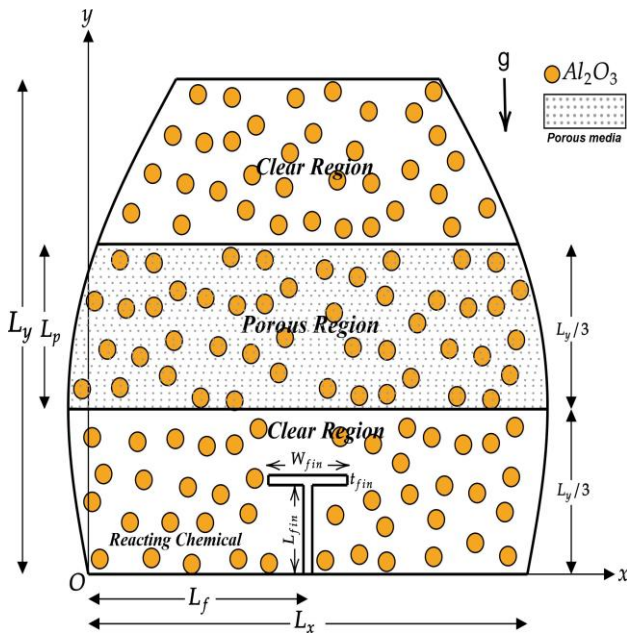


Figure 1: Physical domain of nanofluid and porous layers.

Acting as a passive heat sink, the T-shaped fin disturbs the flow and changes circulation patterns inside the cavity. Meanwhile, Al_2O_3 nanoparticles which are assumed to be chemically inert. However, thermally stable nanoparticles enhance the thermal conductivity of the mixtures. This combination of complex geometric modification and advanced fluid properties (ethanol based nanofluid) is designed to optimize thermal energy dissipation and control thermal runaway during

exothermic chemical reactions. The ethanol fuel and oxidizer mixture is uniform and stable, and treated as a Newtonian fluid. Assuming the Boussinesq approximation for density variations, thermophysical properties remain constant, whereas thermal radiation and viscous dissipation effects are neglected.

The cavity is divided into two clear nanofluid layers ($0 \leq y < \frac{L_y}{3}$) and ($\frac{2L_y}{3} < y \leq L_y$) and a porous layer ($\frac{L_y}{3} \leq y \leq \frac{2L_y}{3}$). In the porous layer, the movement of fluid and heat is described by the generalized Darcy-Brinkman-Forchheimer extended porous model. A homogeneous and isotropic porous medium is assumed.

Considering the above assumptions and following Refs. [43, 44], the governing equations for the combined nanofluid-porous system are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$\begin{aligned} \frac{\rho_{nf}}{\varepsilon^2} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) &= -\frac{\partial p}{\partial x} + \\ \frac{1}{\varepsilon} \left[\frac{\partial}{\partial x} \left(2\mu_{nf} \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left(\mu_{nf} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right) \right] &+ s_x \end{aligned} \quad (3)$$

$$\begin{aligned} \frac{\rho_{nf}}{\varepsilon^2} \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) &= -\frac{\partial p}{\partial y} + \frac{1}{\varepsilon} \left[\frac{\partial}{\partial x} \left(\mu_{nf} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right) + \right. \\ \left. \frac{\partial}{\partial y} \left(2\mu_{nf} \frac{\partial v}{\partial y} \right) \right] &+ s_y \end{aligned} \quad (4)$$

$$\begin{aligned} (\rho c_p)_{nf} \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) &= \frac{\partial}{\partial x} \left(k_{eff} \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k_{eff} \frac{\partial T}{\partial y} \right) + \\ Q \varepsilon c_F c_g A e^{-\frac{E}{RT}} \end{aligned} \quad (5)$$

$$\frac{1}{\varepsilon} \left(u \frac{\partial c_F}{\partial x} + v \frac{\partial c_F}{\partial y} \right) = D_F \left(\frac{\partial^2 c_F}{\partial x^2} + \frac{\partial^2 c_F}{\partial y^2} \right) - \frac{1}{\rho_{nf}} c_F c_g A e^{-\frac{E}{RT}} \quad (6)$$

$$\frac{1}{\varepsilon} \left(u \frac{\partial c_g}{\partial x} + v \frac{\partial c_g}{\partial y} \right) = D_g \left(\frac{\partial^2 c_g}{\partial x^2} + \frac{\partial^2 c_g}{\partial y^2} \right) - \frac{\zeta}{\rho_{nf}} c_F c_g A e^{-\frac{E}{RT}} \quad (7)$$

The equations are used for both the clear regions and the porous region. In clear regions, the porosity (ε) is considered to be 1, while in the porous medium, it is 0.398, which corresponds to glass beads ($d_p = 3\text{mm}$) of thermal conductivity $k_s = 0.845\text{W/m.K}$.

The subscripts nf and eff denote nanofluid and effective medium properties, respectively.

The momentum equations in x – and y – direction have different source terms, s_x and s_y respectively. Equations (8) and (9) are for the clear regions, and equations (10) and (11) are for the porous region.

$$(s_x)_f = 0 \quad (8)$$

$$(s_y)_f = g\rho_{nf}[\beta_T(T_f - T_0) + \beta_{c,F}((c_F)_f - (c_F)_0) + \beta_{c,g}((c_g)_f - (c_g)_0)] \quad (9)$$

$$(s_x)_p = -\frac{\mu_{nf}}{K}u - \frac{F_0}{\sqrt{K}}\rho_{nf}|\vec{V}|u \quad (10)$$

$$(s_y)_p = -\frac{\mu_{nf}}{K}v - \frac{F_0}{\sqrt{K}}\rho_{nf}|\vec{V}|v + g \times \rho_{nf}[\beta_T(T_f - T_0) + \beta_{c,F}((c_F)_f - (c_F)_0) + \beta_{c,g}((c_g)_f - (c_g)_0)] \quad (11)$$

where,

$$|\vec{V}| = \sqrt{u^2 + v^2}, \quad F_0 = \frac{1.75}{\sqrt{150}}\varepsilon^{-3/2}, \quad K = \frac{\varepsilon^3 d_p^2}{150(1 - \varepsilon)^2}$$

The boundary conditions in the dimensional form are:

Here, Initially (when $(t = 0)$)

$$T = T_0, \quad c_F = c_{F0}, \quad c_g = c_{g0} \quad \text{for all } x \text{ and } y.$$

At the left, right and top walls:

$$T = T_0, \quad c_F = c_{F0}, \quad c_g = c_{g0}.$$

At the bottom wall:

$$T = T_0, \quad c_F = c_{F0}, \quad c_g = c_{g0} \quad \text{for } y = 0 \quad \text{and} \quad 0 \leq x \leq L_f - \frac{t_{fin}}{2} \quad \& \quad L_f + \frac{t_{fin}}{2} \leq x \leq L_x.$$

At the fin surface:

$$T = T_0, \quad c_F = c_{F0}, \quad c_g = c_{g0}.$$

At the porous-fluid interface:

$$\text{For } 0 \leq x \leq L_x \text{ and } y = \frac{L_y}{3},$$

$$u = u|_{y=\frac{L_y}{3}} = u|_{y=\frac{L_y}{3}}, \quad v = v|_{y=\frac{L_y}{3}} = v|_{y=\frac{L_y}{3}}, \quad T = T|_{y=\frac{L_y}{3}} = T|_{y=\frac{L_y}{3}},$$

$$c_F = c_F|_{y=\frac{L_y}{3}} = c_F|_{y=\frac{L_y}{3}}, \quad c_g = c_g|_{y=\frac{L_y}{3}} = c_g|_{y=\frac{L_y}{3}};$$

$$\text{also for } 0 \leq x \leq L_x \text{ and } y = \frac{2L_y}{3},$$

$$u = u|_{y=\frac{2L_y}{3}} = u|_{y=\frac{2L_y}{3}}, \quad v = v|_{y=\frac{2L_y}{3}} = v|_{y=\frac{2L_y}{3}}, \quad T = T|_{y=\frac{2L_y}{3}} = T|_{y=\frac{2L_y}{3}},$$

$$c_F = c_F|_{y=\frac{2L_y}{3}} = c_F|_{y=\frac{2L_y}{3}}, \quad c_g = c_g|_{y=\frac{2L_y}{3}} = c_g|_{y=\frac{2L_y}{3}}.$$

No slip condition ($u = v = 0$) is maintained at every wall.

The necessary thermophysical properties of ethanol as the base fluid and the nanoparticles are presented in Table 1 and are assumed to be invariant throughout the simulations.

Abu-Nada [27] presented concise correlations for the viscosity and thermal conductivity of Al_2O_3 -water nanofluid. The correlation is valid within the temperature range ($22^\circ\text{C} - 52^\circ\text{C}$), and the range of nanoparticle volume fraction (Φ) is 1% – 7%.

For the nanofluid viscosity, the relation is

$$\mu_{nf} = \left[\begin{array}{l} -0.1551 - 19.5819\bar{T}^{-1} + 2094.4686\bar{T}^{-2} - 27463.8630\bar{T}^{-3} + 0.7943\Phi \\ -0.1925\Phi^2 + 0.0127\Phi^3 - 8.1121\Phi\bar{T}^{-1} + 1.6044\Phi^2\bar{T}^{-1} + 2.1754\Phi\bar{T}^{-2} \end{array} \right] \quad (12)$$

and, the correlation for thermal conductivity ratio is defined by Chon *et al.* [25]:

$$\frac{k_{nf}}{k_{bf}} = 1 + 64.7 \times \Phi^{0.7460} \times \left(\frac{d_{bf}}{d_{np}}\right)^{0.3690} \times \left(\frac{k_{np}}{k_{bf}}\right)^{0.7476} \times \text{Pr}_{(\bar{T})}^{0.9955} \times \text{Re}_{(\bar{T})}^{1.2321} \quad (13)$$

where,

$$\text{Pr}_{(\bar{T})} = \frac{\mu_{f\bar{T}}}{\rho_{bf}\alpha_{bf}}, \quad \text{Re}_{(\bar{T})} = \frac{\rho_{bf}K_{Bc}\bar{T}}{3\pi\mu_{f\bar{T}}^2\ell_{bf}}, \quad \mu_{f\bar{T}} = 2.414 \times 10^{-5} \times 10^{\frac{247.8}{\bar{T}-140}}. \quad (14)$$

The Boltzmann constant is $K_{Bc} = 1.3807 \times 10^{-23}$ J/K, and the mean free path of the base fluid is $\ell_{bf} = 0.17$ nm. In addition, temperature $\bar{T}$ is measured in degrees Celsius ($^\circ\text{C}$), while dynamic viscosity is given in centipoise (cP).

The following remaining properties of the nanofluid are formulated by Chamkha and Ismael [47]. In the momentum equations (3) and (4), the density of the suspension is obtained as a weighted average of the base fluid and nanoparticles:

Table 1: Thermophysical properties of ethanol and nanoparticles [45, 46]

Material	Type	ρ (kg/m ³)	C_p (J/kg·K)	β (K ⁻¹)	k (W/m·K)	ν (m ² /s)	d (nm)
Ethanol	Base fluid	809.94	3177.87	1.09×10^{-3}	0.18	1.64×10^{-6}	0.446
Al_2O_3	Nanoparticle	3970	765	0.85×10^{-5}	40	–	47

$$\rho_{nf} = (1 - \varphi)\rho_{bf} + \varphi\rho_{np}, \quad (15)$$

where φ indicates the volume fraction of the nanoparticle, and ρ indicates the density.

The subscripts bf and np refer to the base fluid (Ethanol) and the nanoparticle (Al_2O_3) respectively.

Thermal expansion coefficient:

$$(\rho\beta)_{nf} = (1 - \varphi)(\rho\beta)_{bf} + \varphi(\rho\beta)_{np}, \quad (16)$$

Heat capacity:

$$(\rho Cp)_{nf} = (1 - \varphi)(\rho Cp)_{bf} + \varphi(\rho Cp)_{np}, \quad (17)$$

Thermal Diffusivity:

$$\alpha_{nf} = \frac{k_{nf}}{(\rho Cp)_{nf}}, \quad (18)$$

In energy equation (5), the effective thermal conductivity k_{eff} is calculated by:

$$\frac{k_{eff}}{k_{bf}} = (1 - \varepsilon) \frac{k_s}{k_{bf}} + \varepsilon \frac{k_{nf}}{k_{bf}}, \quad (19)$$

where the subscript s indicates the solid matrix.

In this study, the width of the cavity L_x is taken as a reference length for the dimensionless variables. Thus, the following variables are used to reformulate the governing equations and boundary conditions into a dimensionless form:

$$X = \frac{x}{L_x}, \quad Y = \frac{y}{L_x}, \quad U = \frac{uL_x}{\alpha_{bf}}, \quad V = \frac{vL_x}{\alpha_{bf}}, \quad P = \frac{pL_x^2}{\rho_{bf}\alpha_{bf}^2},$$

$$\theta = \frac{(T - T_0)E}{T_0 RT_0}, \quad C_F = \frac{C_F}{C_{F0}}, \quad C_g = \frac{C_g}{C_{g0}}.$$

The dimensionless numbers are defined as follows:

$$\text{Pr} = \frac{\nu_{bf}\rho_{bf}(C_p)_{bf}}{k_{bf}}, \quad H_R = \frac{EQ}{(C_p)_{bf}RT_0^2}, \quad Ra = \frac{g(\beta_T)_{bf}RT_0^2L_x^3}{E\alpha_{bf}\nu_{bf}}, \quad Da = \frac{K}{L_x^2},$$

$$\delta = \frac{A(c_F)_0L_x^2 \exp\left(-\frac{E}{RT_0}\right)}{(\rho)_{bf}\alpha_{bf}}, \quad \gamma = \frac{RT_0}{E}, \quad Le_F = \frac{\alpha_{bf}}{D_F}, \quad Le_g = \frac{\alpha_{bf}}{D_g},$$

$$N_F = \frac{(\rho\beta_{c,F})_{nf}Ec_{F0}}{(\rho\beta)_{bf}RT_0^2}, \quad N_g = \frac{(\rho\beta_{c,g})_{nf}Ec_{g0}}{(\rho\beta)_{bf}RT_0^2}.$$

The parameters are heat of reaction (H_R), Rayleigh number (Ra), Prandtl number (Pr), Darcy number (Da),

Frank-Kamenetski number (δ), Zel'dovich number (γ), Lewis number (Le_F) for ethanol fuel, Lewis number (Le_g) for oxidizer and buoyancy ratio parameter for fuel (N_F) and gas (N_g). The dimensionless stoichiometric ratio (κ) is constant at 1 for this study. These dimensionless parameters describe key features of fluid flow and heat transfer.

By substituting the defined non-dimensional parameters into equations (2) through (11), the dimensionless form of the governing equations for the combined nanofluid-porous system is obtained, which is presented below according to [48]:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (20)$$

$$\frac{1}{\varepsilon^2} \left(\frac{\rho_{nf}}{\rho_{bf}} \right) \left(U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) = - \frac{\partial P}{\partial X} \quad (21)$$

$$+ \frac{Pr}{\varepsilon} \left[\frac{\partial}{\partial X} \left(2 \frac{\mu_{nf}}{\mu_{bf}} \frac{\partial U}{\partial X} \right) + \frac{\partial}{\partial Y} \left(\frac{\mu_{nf}}{\mu_{bf}} \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right) \right) \right] + S_x \quad (22)$$

$$\frac{1}{\varepsilon^2} \left(\frac{\rho_{nf}}{\rho_{bf}} \right) \left(U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} \right) = - \frac{\partial P}{\partial Y} \quad (23)$$

$$+ \frac{Pr}{\varepsilon} \left[\frac{\partial}{\partial X} \left(\frac{\mu_{nf}}{\mu_{bf}} \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right) \right) + \frac{\partial}{\partial Y} \left(\frac{\mu_{nf}}{\mu_{bf}} \frac{\partial V}{\partial Y} \right) \right] + S_y \quad (24)$$

$$\frac{(\rho Cp)_{nf}}{(\rho Cp)_{bf}} \left(U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} \right) = \frac{\partial}{\partial X} \left(\frac{k_{eff}}{k_{bf}} \frac{\partial \theta}{\partial X} \right) + \frac{\partial}{\partial Y} \left(\frac{k_{eff}}{k_{bf}} \frac{\partial \theta}{\partial Y} \right) + \varepsilon H_R \delta C_F C_g e^{\frac{\theta}{(1+\gamma\theta)}} \quad (25)$$

$$\frac{1}{\varepsilon} \left(U \frac{\partial C_F}{\partial X} + V \frac{\partial C_F}{\partial Y} \right) = \frac{1}{Le_F} \left(\frac{\partial^2 C_F}{\partial X^2} + \frac{\partial^2 C_F}{\partial Y^2} \right) - \frac{\rho_{bf}}{\rho_{nf}} \delta C_F C_g e^{\frac{\theta}{(1+\gamma\theta)}} \quad (26)$$

$$\frac{1}{\varepsilon} \left(U \frac{\partial C_g}{\partial X} + V \frac{\partial C_g}{\partial Y} \right) = \frac{1}{Le_g} \left(\frac{\partial^2 C_g}{\partial X^2} + \frac{\partial^2 C_g}{\partial Y^2} \right) - \frac{\rho_{bf}}{\rho_{nf}} \kappa \delta C_F C_g e^{\frac{\theta}{(1+\gamma\theta)}} \quad (27)$$

where, the corresponding source terms are:

$$(S_x)_f = 0 \quad (28)$$

$$(S_y)_f = Pr \times Ra \times \left[\frac{(\rho\beta)_{nf}}{(\rho_{nf}\beta_{bf})} \times \theta + N_F(C_F - 1) + N_g(C_g - 1) \right] \quad (29)$$

$$(S_x)_p = - \frac{\mu_{nf}}{\mu_{bf}} \times \frac{Pr}{Da} U - \frac{\rho_{nf}}{\rho_{bf}} \times \frac{1.75}{\sqrt{150}} \times \frac{1}{\sqrt{\varepsilon}\sqrt{Da}} U \sqrt{U^2 + V^2} \quad (30)$$

$$(S_y)_p = - \frac{\mu_{nf}}{\mu_{bf}} \times \frac{Pr}{Da} V - \frac{\rho_{nf}}{\rho_{bf}} \times \frac{1.75}{\sqrt{150}} \times \frac{1}{\sqrt{\varepsilon}\sqrt{Da}} V \sqrt{U^2 + V^2}$$

$$+ Pr \times Ra \times \left[\frac{(\rho\beta)_{nf}}{(\rho_{nf}\beta_{bf})} \times \theta + N_F(C_F - 1) + N_g(C_g - 1) \right] \quad (31)$$

The boundary conditions can be written as dimensionless forms as follows:

At the left, right and top walls:

$$\theta = 0, \quad C_F = 1, \quad C_g = 1.$$

At the bottom wall ($Y = 0$):

$$\theta = 0, \quad C_F = 1, \quad C_g = 1 \quad \text{for} \quad \text{and} \quad 0 \leq X \leq L_F - \frac{t_{Fin}}{2} \quad \& \quad L_F + \frac{t_{Fin}}{2} \leq X \leq A.$$

At the fin surface:

$$\theta = 0, \quad C_F = 1, \quad C_g = 1.$$

At the interface walls ($Y = \frac{1}{3}$ and $Y = \frac{2}{3}$):

$$U = U|_{Y^-} = U|_{Y^+}, \quad V = V|_{Y^-} = V|_{Y^+}, \quad \theta = \theta|_{Y^-} = \theta|_{Y^+},$$

$$C_F = C_F|_{Y^-} = C_F|_{Y^+}, \quad C_g = C_g|_{Y^-} = C_g|_{Y^+}.$$

Here $A = L_y/L_x = 1$ is the aspect ratio of the enclosure. No slip condition ($U = V = 0$) is maintained at every wall.

To capture the internal flow structure of the enclosure, streamlines are used, based on the solution of the stream function Poisson equation, and are expressed in the following form:

$$\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right) = -\left(\frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}\right) \quad (32)$$

3. THERMODYNAMIC ANALYSIS OF ENTROPY GENERATION

In accordance with the second law of thermodynamics, convective heat transfer, the transport of environmental air into the system, and its subsequent exothermic chemical reaction with the ethanol-based nanofluid are fundamentally dissipative phenomena. These irreversible processes continuously generate entropy. The total entropy generation represents the energy dissipated by different transport processes within the system. Consider a generalized system with n components. The entropy generation rate is expressed as the sum of the products of the thermodynamic fluxes (I_j) and their corresponding driving forces (F_j). This relationship is given by:

$$S_{gen} = \sum_{j=1}^n I_j F_j. \quad (33)$$

In the context of double diffusive convection, the volumetric entropy generation rate is strictly governed by heat transfer, mass diffusion, fluid friction, and chemical reaction. The secondary Soret and Dufour cross diffusion effects are neglected under current

operating conditions. Consequently, the total entropy generation is evaluated as the sum of irreversible effects induced by thermal gradients (S_T), concentration gradients (S_D), and viscous dissipation (S_μ). When analyzing the fluid dynamics within a partially porous cavity, the Darcy effect term acts as a critical contributor to viscous irreversibility and must not be neglected relative to clear fluid viscous dissipation. The local dimensional volumetric entropy generation rate (S_{gen}) can be expressed as [49]:

$$S_{gen} = S_T + S_\mu + S_D, \quad (34)$$

where, entropy due to thermal gradients (S_T) in clear layers and porous layer:

$$S_T = \left(\frac{(1-\varepsilon)k_s + \varepsilon k_{nf}}{T_0^2}\right) \times \left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] \quad (35)$$

The equations are used for both the clear layers ($S_T|_f$) and the porous layer ($S_T|_p$). In clear layers, the porosity (ε) is considered to be 1, while in the porous layer, it is 0.398.

Entropy due to viscous dissipation (S_μ):

$$S_\mu = S_\mu|_f + S_\mu|_p \quad (36)$$

where, $S_\mu|_f$ and $S_\mu|_p$ are the entropy generation due to viscous irreversibility within the clear layers and porous layer, respectively:

$$S_\mu|_f = \frac{\mu_{nf}}{T_0} \left[2 \left(\frac{\partial u}{\partial x}\right)^2 + 2 \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \right], \quad (37)$$

$$S_\mu|_p = \frac{\mu_{nf}}{T_0} \left[2 \left(\frac{\partial u}{\partial x}\right)^2 + 2 \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \right] + \frac{\mu_{nf}}{T_0 K} (u^2 + v^2). \quad (38)$$

Entropy due to concentration gradients (S_D) in clear layers ($S_D|_f$) and porous layer ($S_D|_p$):

$$S_D = \frac{RD_F}{c_{F0}} \left[\left(\frac{\partial c_F}{\partial x}\right)^2 + \left(\frac{\partial c_F}{\partial y}\right)^2 \right] + \frac{RD_F}{T_0} \left[\left(\frac{\partial c_F}{\partial x}\right) \left(\frac{\partial T}{\partial x}\right) + \left(\frac{\partial c_F}{\partial y}\right) \left(\frac{\partial T}{\partial y}\right) \right] + \frac{RD_D}{c_{g0}} \left[\left(\frac{\partial c_g}{\partial x}\right)^2 + \left(\frac{\partial c_g}{\partial y}\right)^2 \right] + \frac{RD_D}{T_0} \left[\left(\frac{\partial c_g}{\partial x}\right) \left(\frac{\partial T}{\partial x}\right) + \left(\frac{\partial c_g}{\partial y}\right) \left(\frac{\partial T}{\partial y}\right) \right]. \quad (39)$$

Applying the dimensionless transformations defined above, the local dimensionless entropy generation rate is formulated as follows:

$$S_{total} = S_\theta + S_F|_f + S_F|_p + S_C, \quad (40)$$

where the dimensionless components are strictly defined as [38, 50]:

$$S_\theta = \left((1 - \varepsilon) \frac{k_s}{k_{bf}} + \varepsilon \frac{k_{nf}}{k_{bf}} \right) \times \left[\left(\frac{\partial \theta}{\partial X} \right)^2 + \left(\frac{\partial \theta}{\partial Y} \right)^2 \right] \quad (41)$$

$$S_F|_f = \Pi_1 \left(\frac{\mu_{nf}}{\mu_{bf}} \right) \left[2 \left(\frac{\partial U}{\partial X} \right)^2 + 2 \left(\frac{\partial V}{\partial Y} \right)^2 + \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right)^2 \right] \quad (42)$$

$$S_F|_p = \Pi_1 \left(\frac{\mu_{nf}}{\mu_{bf}} \right) \left[2 \left(\frac{\partial U}{\partial X} \right)^2 + 2 \left(\frac{\partial V}{\partial Y} \right)^2 + \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right)^2 + \frac{1}{Da} (U^2 + V^2) \right] \quad (43)$$

$$S_C = \Pi_2 \left[\left(\frac{\partial C_F}{\partial X} \right)^2 + \left(\frac{\partial C_F}{\partial Y} \right)^2 \right] + \Pi_3 \left[\left(\frac{\partial C_F}{\partial X} \right) \left(\frac{\partial \theta}{\partial X} \right) + \left(\frac{\partial C_F}{\partial Y} \right) \left(\frac{\partial \theta}{\partial Y} \right) \right] + \Pi_4 \left[\left(\frac{\partial C_g}{\partial X} \right)^2 + \left(\frac{\partial C_g}{\partial Y} \right)^2 \right] + \Pi_5 \left[\left(\frac{\partial C_g}{\partial X} \right) \left(\frac{\partial \theta}{\partial X} \right) + \left(\frac{\partial C_g}{\partial Y} \right) \left(\frac{\partial \theta}{\partial Y} \right) \right] \quad (44)$$

Here, $\Pi_1, \Pi_2, \Pi_3, \Pi_4$, and Π_5 are the irreversibility dimensionless distribution parameters that scale the thermodynamic impact of fluid friction, and mass diffusion of both reacting species.

They are defined as: $\Pi_1 = \frac{\mu_{bf} T_0}{2k_{bf}} \left(\frac{\alpha_{bf}^2}{L_x^2 (\Delta T)^2} \right)$, $\Pi_2 = \frac{RD_F T_0}{k_{bf} c_{F0}} \left(\frac{\Delta C_F}{\Delta T} \right)^2$, $\Pi_3 = \frac{RD_F}{k_{bf}} \left(\frac{\Delta C_F}{\Delta T} \right)$, $\Pi_4 = \frac{RD_g T_0}{k_{bf} c_{g0}} \left(\frac{\Delta C_g}{\Delta T} \right)^2$, $\Pi_5 = \frac{RD_g}{k_{bf}} \left(\frac{\Delta C_g}{\Delta T} \right)$.

According to Mondal *et al.* [51], $\Pi_1 = 0.0001$, $\Pi_2 = \Pi_4 = 0.5$, and $\Pi_3 = \Pi_5 = 0.01$. The final form of the global entropy generation may be evaluated from [52]:

$$S_{global} = S_\theta dXdY + S_F|_f dXdY + S_F|_p dXdY + S_C dXdY. \quad (45)$$

To precisely quantify the specific contribution of the partially porous layer cavity to the total thermodynamic irreversibility of the system, an entropy generation ratio (EGR) is introduced. This parameter is defined as the ratio of the entropy generated strictly within the porous domain to the global entropy generation [53]:

$$EGR = \frac{S_{por} dXdY}{S_{global}}, \quad (46)$$

where,

$$S_{por} = S_\theta|_p + S_F|_p + S_C|_p. \quad (47)$$

A more comprehensive assessment of these thermodynamic irreversibilities is achieved by introducing the Bejan number (Be). It represents the ratio of entropy generated by heat transfer, multi component mass diffusion, and the chemical reaction to the total entropy generation. As defined in [54, 55], this dimensionless parameter is expressed as:

$$Be = \frac{S_\theta dXdY + S_C dXdY}{S_{global}}. \quad (48)$$

The local Bejan number (Be) is strictly bounded within the closed interval $[0,1]$. A value of $Be = 1$ indicates that entropy generation is entirely due to heat transfer, whereas $Be = 0$ represents irreversibility caused only by fluid friction and porous media resistance. At $Be = 0.5$, both mechanisms contribute equally to entropy generation. In the context of the present system, a value of $Be > 0.5$ dictates that convective heat transfer, mass diffusion, and the exothermic chemical reaction serve as the primary sources of irreversibility. In contrast, $Be < 0.5$ identifies a strictly friction dominated flow regime, this effect is primarily observed within the partially porous layer and near the solid boundaries.

4. NUMERICAL METHODOLOGY AND VALIDATIONS

The dimensionless governing equations and entropy generation equations are solved using the Galerkin finite element method (GFEM). The computational domain is discretized using free triangular elements. A extremely fine mesh is employed near the T-shaped fin, wavy boundaries, and reaction zones to accurately capture the temperature and velocity gradients. Initially, the governing partial differential equations are transformed into their corresponding weak (variational) formulations. These weak forms are subsequently integrated over the discretized computational domains to establish a system of residual equations. The resulting nonlinear algebraic system is iteratively solved utilizing the PARDISO direct solver in conjunction with the Newton-Raphson method. Within each finite element, the continuous dependent variables specifically the velocity components (U, V), concentrations (C_F, C_g), and temperature (θ) (represented generally as W) are expressed as a linear combination of the shape functions Ψ_j :

$$W = \sum_{j=1}^m W_j \Psi_j(X, Y). \quad (49)$$

In accordance with the Galerkin weighted residual framework, the weight functions are chosen to be identical to the shape functions. To resolve the nonlinearities within the governing equations, the Newton-Raphson linearization scheme is applied in the following iterative form:

$$\zeta_{j+1} = \zeta_j - \frac{g(\zeta_j)}{g'(\zeta_j)} \quad (50)$$

Convergence of the iterative numerical procedure is achieved when the relative error between consecutive

iterations for all dependent variables falls below a strictly defined tolerance threshold:

$$\left| \frac{\zeta_{j+1} - \zeta_j}{\zeta_{j+1}} \right| \leq 10^{-6} \quad (51)$$

To ensure the results were independent of the grid size, a mesh sensitivity study was conducted using eight different element densities with the parameters: $Ra = 10^4$, $\phi = 0.05$, $Da = 10^{-3}$, $H_R = 5$, $\delta = 10$, $Le_F = 0.1$, $Le_g = 1$, $N_F = 0.5$, $N_g = 1$. Comparing the maximum temperature, and maximum stream function (Table 2), it is noticed that the mesh with 55,083 elements achieved sufficient convergence. This table illustrates how θ_{max} , and ψ_{max} vary across different element counts. Based on these results, the finalized mesh configuration consists of 55,083 free triangular elements, ensuring high numerical accuracy for the simulation.

To ensure the reliability of the present numerical solver, the results were validated against Al-Zamily [56] regarding the natural convection and entropy generation of a nanofluid in the presence of a discrete heat source within a multilayer partially porous layer cavity. The isotherms (θ) were computed for Case-1 using $Ra = 10^5$, $Da = 10^{-3}$, $L_p = L/3$, $\phi = 0.1$, and $\lambda = 0$. Similarly, the streamfunctions (ψ) for Case-2 were evaluated using the corresponding values of $Ra = 10^7$, $Da = 10^{-5}$, $L_p = L/3$, $\phi = 0.1$, and $\lambda = 0$. The resulting fluid flow and thermal patterns demonstrate excellent agreement with previously published results. Our results showed excellent similarity with the flow patterns and thermal gradients, as observed in Fig. (2). These results validate the ability of the present method to accurately predict natural convection characteristics within a partially porous layered enclosure.

As illustrated in Fig. (3), the predicted isotherms and streamlines exhibit very close resemblance to the isotherms of the present study and the streamlines using numerical data of Beckermann *et al.* [18] from experiment 12, configuration 2. Quantitatively, the predicted central isotherm ($\theta = 0.5$) demonstrates an exact match with the published data. For the fluid flow, the maximum streamfunction (ψ_{max}) of the present study shows a maximum deviation of 8.78% in the left vortex (yielding 13.5 compared to the published value of 14.8) and a 6.08% deviation in the right vortex (yielding 13.9).

The temperature profiles are validated against experimental data in Fig. (4) for experiment 3, configuration 2. The data used for this validation case are $Da = 2.248 \times 10^{-6}$, $Ra = 9.52 \times 10^5$, $Pr = 8356.2$, $C = 0.565$, and $R_k = 2.259$. Quantitative analysis of the extracted data points reveals an average error of approximately 3.5%, with a maximum local deviation of 7.9% occurring near the right boundary.

The results clearly indicate that the present method achieves a satisfactory level of accuracy when compared with Beckermann *et al.* [18] numerical investigations. However, a slight discrepancy exists between the experimental and numerical results, a trend also noted in the original study by Beckermann *et al.* [18]. This difference can be explained by the presence of experimental uncertainties. Although minor variations are observed, the observed agreement indicates that the present model is reliable for further investigations.

Furthermore, entropy and Bejan number (Be) were quantitatively compared against the results of Al-Zamily [56] across various values of volume fraction(ϕ), keeping $\lambda = 0$, $Pr = 6.44$, $Ra = 10^4$, $Da = 10^{-1}$ constant for Case-1 in Table 3.

Table 2: Grid Study for θ_{max} , and ψ_{max} with Consecutive Relative Errors

Mesh	No. of elements	θ_{max}	Error(%)	$ \psi_{max} $	Error(%)
1	958	1.15266	-	0.43568	-
2	1,651	1.14652	0.533	0.44386	1.878
3	3,782	1.14464	0.164	0.44936	1.239
4	7,405	1.14591	0.111	0.45146	0.467
5	14,822	1.14660	0.060	0.45293	0.326
6	27,663	1.14673	0.011	0.45352	0.130
7	55,083	1.14697	0.021	0.45379	0.060
8	1,11,861	1.14713	0.014	0.45407	0.062

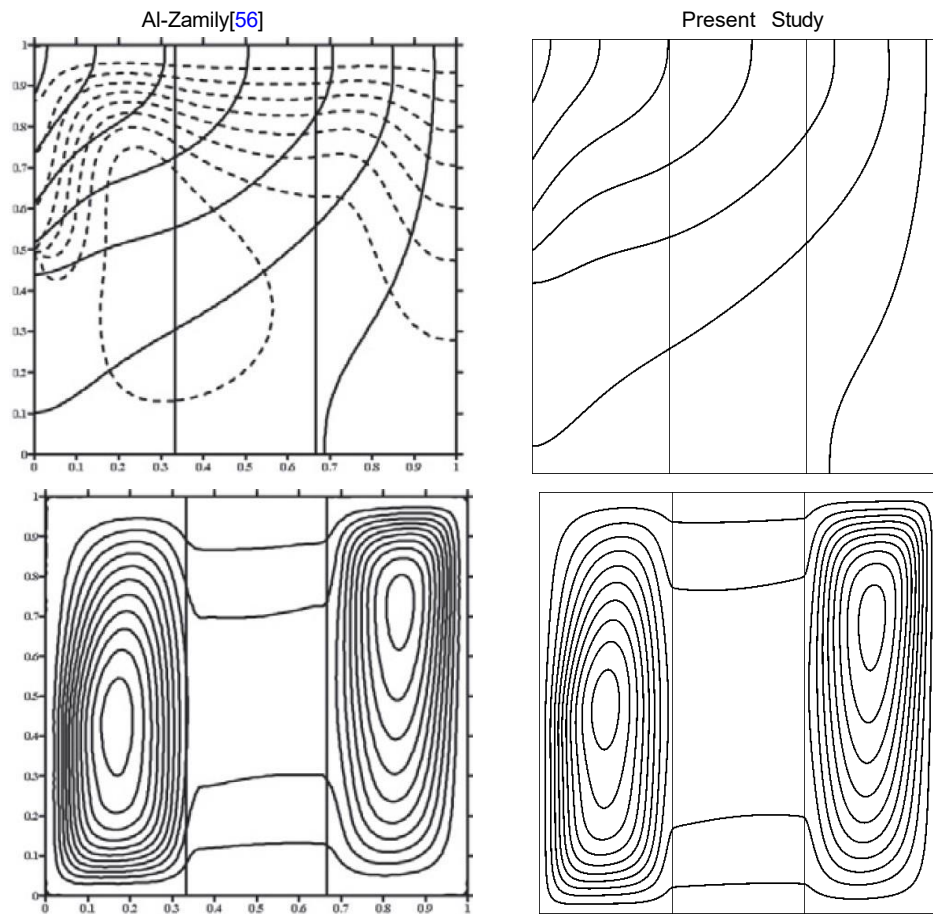


Figure 2: Comparison of isotherms and streamfunctions between the current study and Al-Zamily[56].

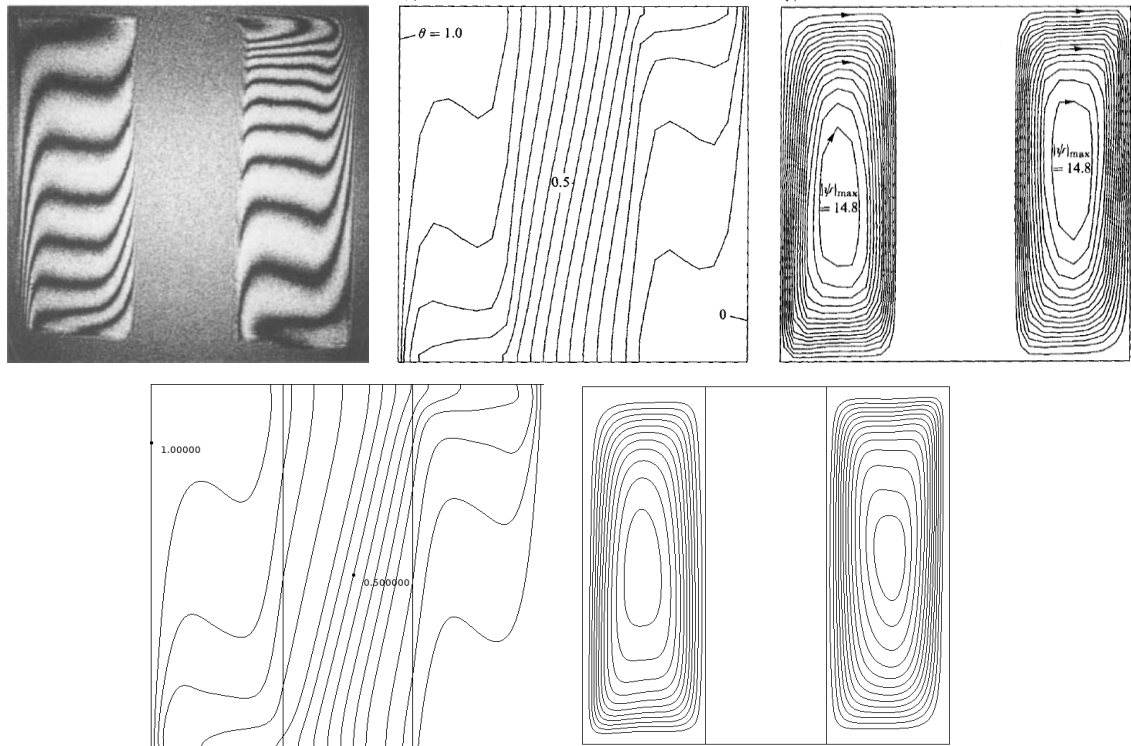


Figure 3: Comparison of isotherms and streamlines between present study and Beckermann *et al.* [18] (configuration 2, experiment 11) at $Ra = 3.7 \times 10^6$, $Pr = 6.44$, $Da = 7.242 \times 10^{-5}$, $Rk = 1.362$ and $C = 0.554$.

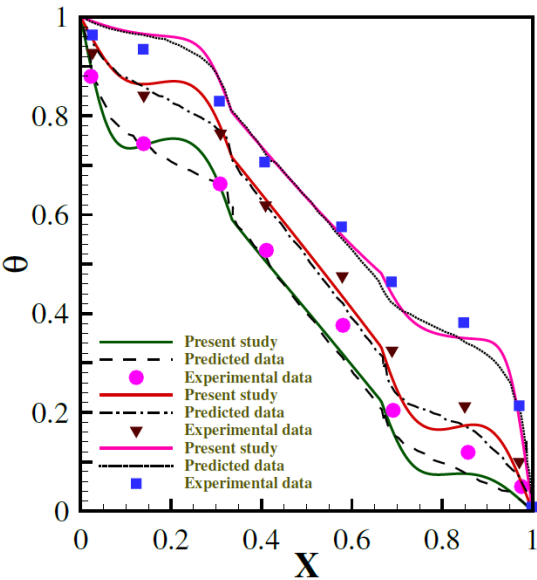


Figure 4: Validation of experimental and predicted temperature profiles with Beckermann *et al.* [18] for configuration 2, experiment 3.

Table 3: Comparison of Present Solution with the Result of Al-Zamily[56] for Bejan Number (Be)

φ	<i>Be</i>		φ	<i>Be</i>	
	Al-Zamily [56]	This Study		Al-Zamily [56]	This Study
.00	0.022968	0.022767	0.06	0.029152	0.029078
.01	0.023001	0.022994	0.07	0.030035	0.030026
.02	0.023852	0.023477	0.08	0.031802	0.031796
.03	0.025618	0.025431	0.09	0.032086	0.031961
.04	0.026502	0.026493	0.095	0.032686	0.032596
.05	0.027385	0.027162	0.10	0.034452	0.034436

5. RESULT AND DISCUSSION

This study presents a comprehensive analysis of isotherms, streamfunctions, isoconcentra-tions, and entropy generation under the combined effects of fluid friction, heat transfer, and species transport during the natural convection of a chemically reacting ethanol-based (Al_2O_3) nanofluid within a partially porous dome-shaped cavity equipped with a T-shaped fin. Owing to its geometric configuration, a dome-shaped enclosure is anticipated to enhance convective fluid motion; therefore, the present investigation aims to elucidate the influence of dome geometry on the natural convection behavior of the ethanol-based (Al_2O_3) nanofluid.

The thermal and thermodynamic performance of the system is evaluated through a comparative assessment of the Bejan number (Be), entropy generation ratio (EGR), and dimensionless global entropy generation

(S_{global}) under varying governing parameters. The main objective of this study is to identify suitable conditions for minimizing entropy generation to optimize thermodynamic performance.

The simulations were conducted by varying the parameter values within the defined ranges Rayleigh number ($10^3 \leq Ra \leq 10^6$), Darcy number ($10^{-5} \leq Da \leq 10^{-2}$), Frank–Kamenetskii number ($5 \leq \delta \leq 20$), Lewis number ($0.1 \leq Le_F \leq 15$, $1 \leq Le_g \leq 10$), nanoparticle volume fraction ($0.01 \leq \phi \leq 0.07$), Bouancy paramater ratio ($0.1 \leq N_F, N_g \leq 10$), porous layer thickness ($0.1 \leq L_P \leq 0.5$), and T-shaped fin length ($0.13 \leq L_{Fin} \leq 0.23$).

Thermal fields are qualitatively interpreted via isotherms to identify hotspots (θ_{max}), while flow dynamics are evaluated through streamlines, specifically focusing on the maximum stream function value (ψ_{max}) to determine the strength of the circulation cells.

Additionally, Species transport of ethanol fuel and oxidizer are illustrated through concentration profiles $(C_F)_{min}$, $(C_g)_{min}$ respectively.

Through this systematic parametric study, the individual and combined roles of fin placement, porous permeability, and nanofluid concentration are determined to optimize the thermal performance and heat dissipation of the system.

Unless otherwise specified, the constants and governing parameters are fixed as $\gamma = 0.2$, $\varepsilon = 0.398$, $k_s = 0.845$, $L_p = \frac{1}{3}$, $L_{Fin} = 0.18$, $W_{Fin} = 0.18$, $t_{Fin} = 0.18$, $H_R = 5$, $\delta = 10$, $Le_F = 5$, $Le_g = 10$, $N_F = 2$, $N_g = 1$, $\phi = 0.05$, $Da = 10^{-3}$, and $Ra = 10^4$.

Figure 5 shows how decreasing the Darcy number ($10^{-5} \leq Da \leq 10^{-2}$) restricts the fluid flow, heat, and species transport inside the cavity (keeping $Ra = 10^4$ and $\phi = 0.02$ constant). The Darcy number represents

the permeability of the porous layer. A higher Da means a less dense porous structure, offering less resistance to flow. Thus, for higher values of Da , the fluid flows through the porous layer relatively freely. When the permeability is high ($Da = 10^{-2}$), buoyancy overcomes viscous resistance and produces a strong convective flow ($\psi_{max} \approx 0.711$). This creates four symmetric recirculating vortices that fill the entire cavity. They promote the formation of thermal plumes ($\theta_{max} \approx 0.493$). They also enhance species transport near the T-fin sidewalls, with $(C_F)_{min} \approx 0.531$ and $(C_g)_{min} \approx 0.261$. However, as the value of Da decreases, the porous layer becomes more restrictive. It corresponds to a denser structure, where the solid matrix occupies a greater fraction of the medium. The augmented Darcy drag force within this dense matrix markedly slows the fluid motion, reducing the convective strength by approximately 43.1% at $Da = 10^{-5}$ ($\psi_{max} \approx 0.404$). This added resistance completely changes the streamline patterns. Fluid circulation stops within the

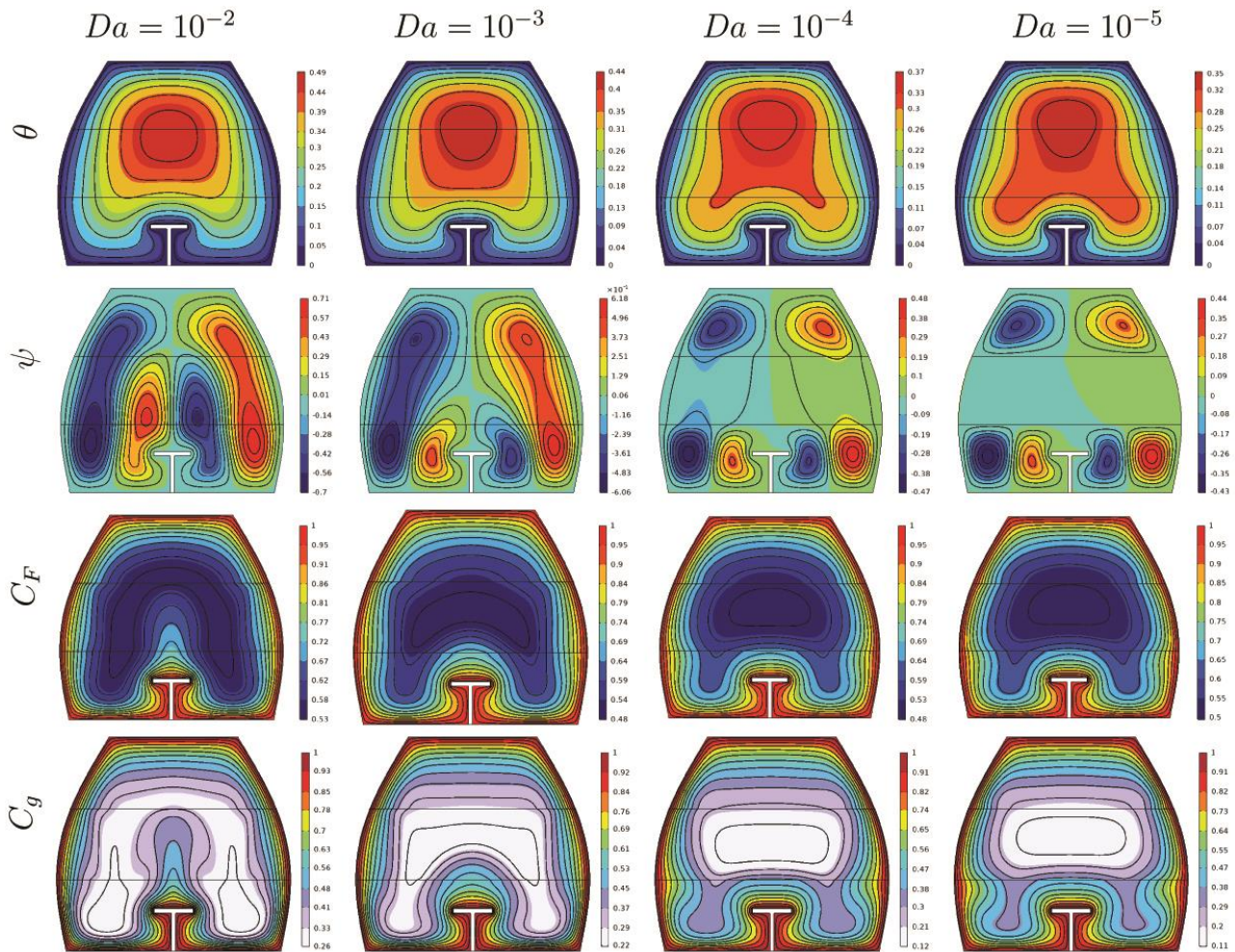


Figure 5: Contours of isotherms (θ), streamlines (ψ), fuel concentration (C_F), and gas concentration (C_g) for different Da at $\phi = 0.02$, $Ra = 10^4$.

upper porous layer, so no vortices form there. Instead, localized vortex pairs develop only in the clear fluid regions above the porous layer and near the T-shaped fin. Because convection weakens, the system shifts to a conduction and diffusion dominated state. This shows in the isotherms, which become highly stratified with a lower peak temperature ($\theta_{max} \approx 0.35$) for $Da = 10^{-5}$, while the concentration boundary layers thicken, indicating diffusion limited transport. The centers of the isotherms and concentration isolines migrate upward from the bottom of the cavity as the high temperature regions spread to envelop the porous layer.

Figures 6 presents the influence of Ra and ϕ across $10^3 \leq Ra \leq 10^6$, and $0.02 \leq \phi \leq 0.05$, evaluated under fixed parametric values: $Da = 10^{-3}$, $\delta = 10$, $Le_F = 2$, $Le_g = 3$, $H_R = 3$, and $N_F = N_g = 1$.

The subplots show how the Rayleigh number and nanoparticle volume fraction determine the fluid flow, thermal, and species transfer characteristics. At lower Rayleigh numbers ($Ra \leq 10^4$), buoyancy forces remain weak, and heat transfer is dominated by conduction, resulting in low minimum species concentrations ($(C_F)_{min}$ and $(C_g)_{min}$). As Ra increases to 10^6 ,

buoyancy forces overcome viscous resistance, making convection dominant. The stream function magnitude ($|\psi_{max}|$) increases by approximately five orders of magnitude, reaching $|\psi_{max}| \approx 5.49$. This strong circulation mixes the species throughout the partially porous layer cavity and increases their minimum concentrations significantly, reaching approximately 0.74 for $(C_F)_{min}$ and 0.67 for $(C_g)_{min}$.

The thermal response is nonmonotonic; the maximum temperature (θ_{max}) increases up to $Ra = 10^5$ but decreases at $Ra = 10^6$. This drop at elevated Rayleigh numbers occurs because the highly intense advection and the consequent thinning of the thermal boundary layer lead to rapid heat dissipation, overriding the internal heat generation effects. This strong convective flow distributes heat more uniformly and eliminates localized hot spots. Meanwhile, adding nanoparticles introduces two competing effects. Increasing ϕ lowers the maximum temperature by improving the effective thermal conductivity of the fluid. However, it reduces the maximum flow strength ($|\psi_{max}|$) at higher Ra due to the augmented dynamic viscosity of the nanofluid. Nanoparticle addition also affects species transport. For any given Ra , increasing

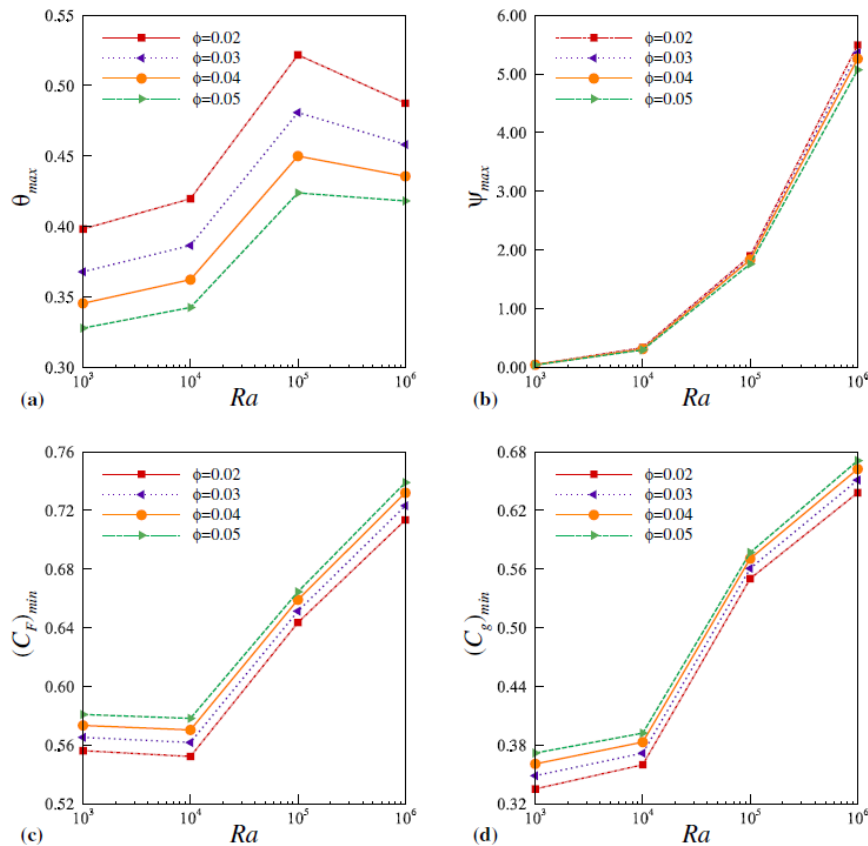


Figure 6: Influence of Ra and ϕ on θ_{max} , ψ_{max} , $(C_F)_{min}$, and $(C_g)_{min}$ for $H_R = 3$, $N_F = 1$, $Le_F = 2$, $Le_g = 3$.

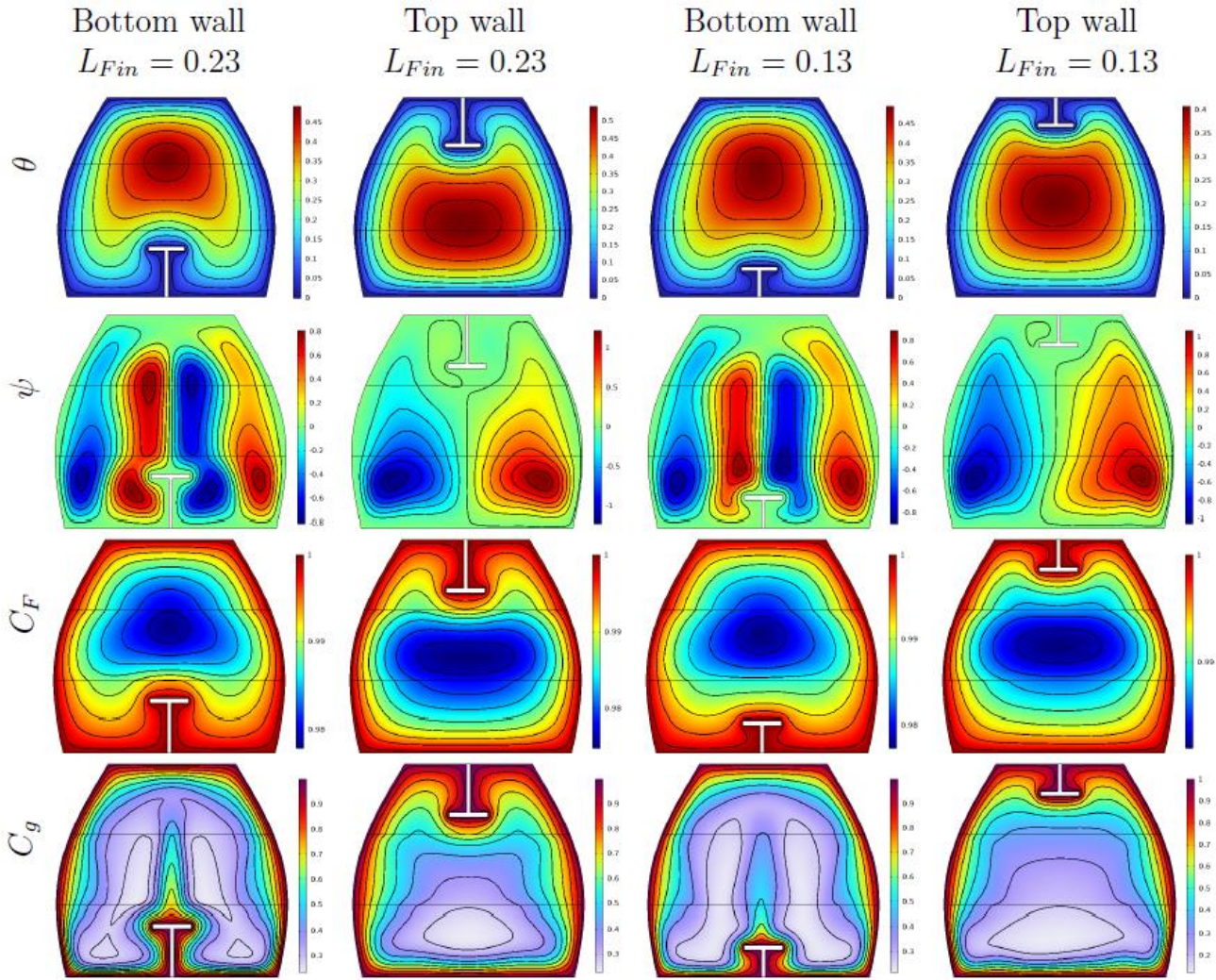


Figure 7: Contours of isotherms (θ), streamlines (ψ), fuel concentration (C_F), and gas concentration (C_g) for different fin position and length (L_{Fin}) at $Ra = 10^5$, $\phi = 0.07$, $L_{eF} = 0.1$.

ϕ increases the minimum species concentrations, with both $(C_F)_{min}$ and $(C_g)_{min}$ rising as ϕ increases. Therefore, a higher nanoparticle volume fraction improves the overall species distribution within the cavity.

The internal geometry of the partially porous layer cavity heavily dictates transport processes, specifically through the physical constraints imposed by the T-shaped fin's placement and length (L_{Fin}) alongside the porous layer thickness (L_P). Figures 7 and 8 demonstrate how these barriers alter flow configurations, as well as the distributions for heat and species transport.

Figure 7 presents the effects of the T-shaped fin placement (bottom or top wall) and its length ($L_{Fin} = 0.13$ and 0.23). The results are obtained at $Ra = 10^5$ and $\phi = 0.07$. As a mechanical barrier, the fin controls

the stream function (ψ) of fluid flow. Mounting it on the bottom wall breaks up the rising flow into a complex structure with multiple recirculating vortices. When the fin is on the top wall, the restriction stays at the top boundary, allowing two dominant vortices to form in the rest of the cavity. Naturally, the longer fin ($L_{Fin} = 0.23$) creates a much stronger blockage, distorting the flow and thermal fields far more severely than the shorter fin ($L_{Fin} = 0.13$). Because the extended fin takes up more space, it traps more fluid near its surfaces and creates larger stagnation regions where the flow barely moves. This drop in local velocity weakens the convection process, making it noticeably harder for the heat and species to mix uniformly. This hydrodynamic reorganization directly governs the thermal and concentration distributions. With a bottom-mounted fin, the central high temperature core (θ_{max}) and the primary low concentration of the fuel ($(C_F)_{min}$) are

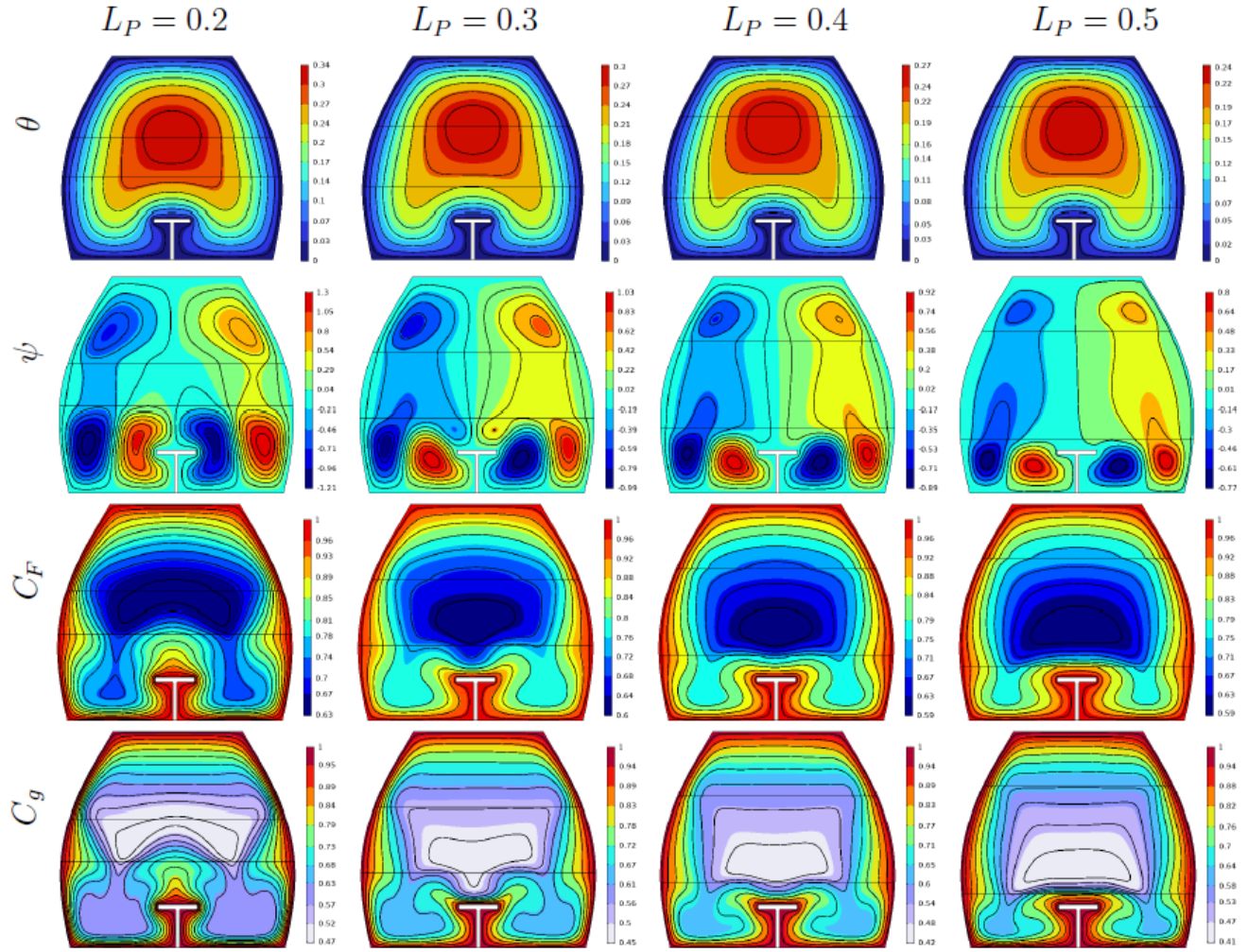


Figure 8: Contours of isotherms (θ), streamlines (ψ), fuel concentration (C_F), and gas concentration (C_g) for various porous layer thickness (L_P) at $\phi = 0.06$, $Ra = 10^5$, $\delta = 5$, $N_g = 0.5$, $Da = 10^{-4}$.

physically pushed upward into the top half of the enclosure. This bottom placement also triggers distinct vertical, plume-shaped distributions in the gas concentration (C_g) positioned symmetrically on either side of the fin. Relocating the fin to the top wall forces these regions downward, spreading them horizontally across the midsection and suppressing localized vertical plumes.

Figure 8 examines the effect of increasing the porous layer thickness ($0.2 \leq L_P \leq 0.5$) under fixed low permeability conditions ($Da = 10^{-4}$). The figure shows how the expanded porous layer influences the transport processes within the cavity. Low Darcy numbers increase internal flow resistance, so fluid circulates in a multicellular vortex structure only when the porous layer is thin ($L_P = 0.2$). As the porous layer

thickens, the available flow region shrinks, restricting circulation in the upper cavity. This viscous drag forces convective flow into the clear fluid space at the bottom, flattening the upper vortices and reducing flow intensity. Because the thicker porous zone introduces more solid material into the cavity, it physically blocks the fluid from carrying heat and chemical species to the upper regions. This limits upward convection and species transport, displacing the high temperature core (θ) downward. Similarly, θ_{max} decreases as the growing porous zone makes the cavity conduction dominated. The species concentration fields (C_F and C_g) follow this trend; their central cores lose vertical reach and flatten near the bottom wall, indicating weaker mixing as the thicker porous layer shifts the upper portion into a diffusion-dominated regime.

Figure (9) illustrates the effects of the Frank-Kamenetskii number (δ) and the nanoparticle volume fraction (ϕ). The results are evaluated at a constant $Ra = 10^4$ and $Da = 10^{-3}$. Their influence on the peak temperature, maximum flow strength, and minimum species concentrations is presented. The Frank-Kamenetskii number (δ) is a key dimensionless parameter in reactive fluid flows, characterizing the ratio of heat generated by exothermic chemical reactions to heat removed by conduction. Increasing the Frank-Kamenetskii number ($5 \leq \delta \leq 20$) improves the heat released by the exothermic reaction. This extra heat raises the maximum temperature (θ_{max}), which in turn strengthens the local buoyancy forces and drives a faster fluid flow (ψ_{max}).

However, adding more nanoparticles into the base fluid consistently counters these increases. As ϕ rises from 0.01 to 0.07, the enhanced effective thermal conductivity of the nanofluid dissipates heat more efficiently, clearly reducing the peak temperature (θ_{max}). Simultaneously, a higher nanoparticle

concentration makes the fluid more viscous. The increased viscosity creates greater flow resistance, reducing the maximum flow strength across the entire range of δ .

Species transport reveals a distinct, opposing pattern. As δ increases, the intensified exothermic reaction consumes more species. The stronger convective flow also enhances species transport. This reduces both the minimum fuel ($(C_F)_{min}$) and gas ($(C_g)_{min}$) concentrations. The addition of nanoparticles affects these two species in opposite ways. For the fuel, increasing ϕ slightly raises $(C_F)_{min}$ because the higher viscosity slows the flow, preventing the fuel from washing out completely. On the other hand, a higher ϕ lowers $(C_g)_{min}$, dropping the green dashed line ($\phi = 0.07$) below the red solid line ($\phi = 0.01$). These opposite trends happen because the fuel and gas have different diffusion rates (Lewis numbers). When the thicker nanofluid slows down the overall flow, the fuel tends to build up slightly in the cavity, while the gas continues to react and deplete more rapidly in the heated zones.

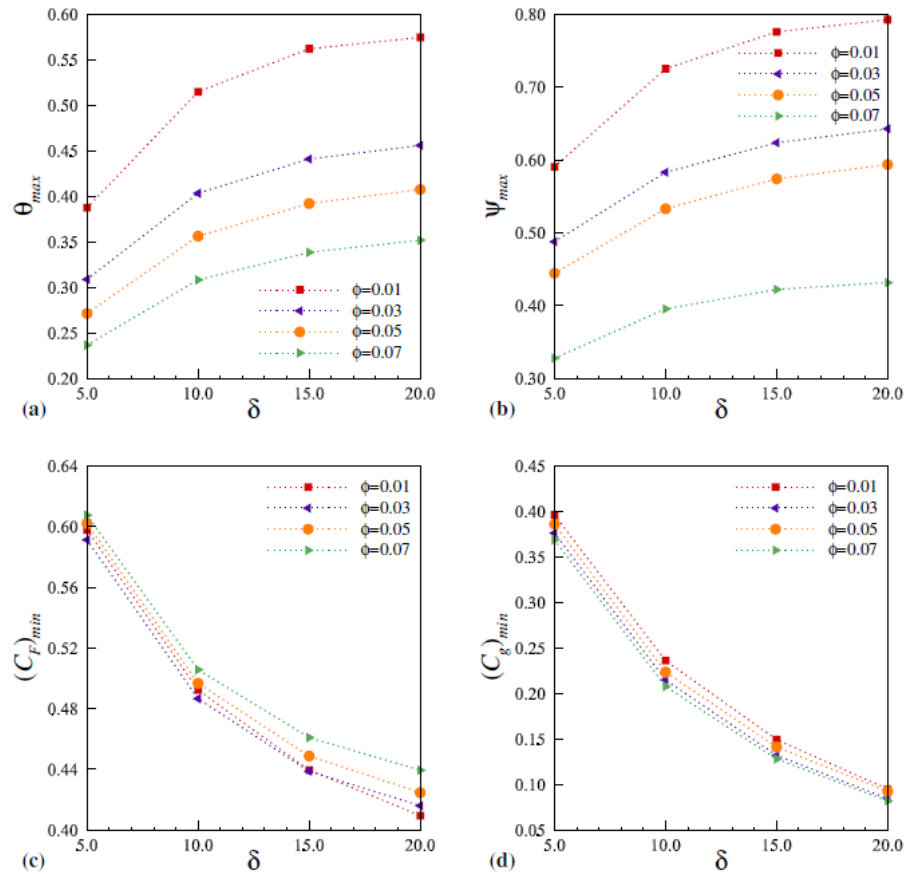


Figure 9: Influence of δ and ϕ on θ_{max} , ψ_{max} , $(C_F)_{min}$, and $(C_g)_{min}$ for $Ra = 10^4$, $Da = 10^{-3}$.

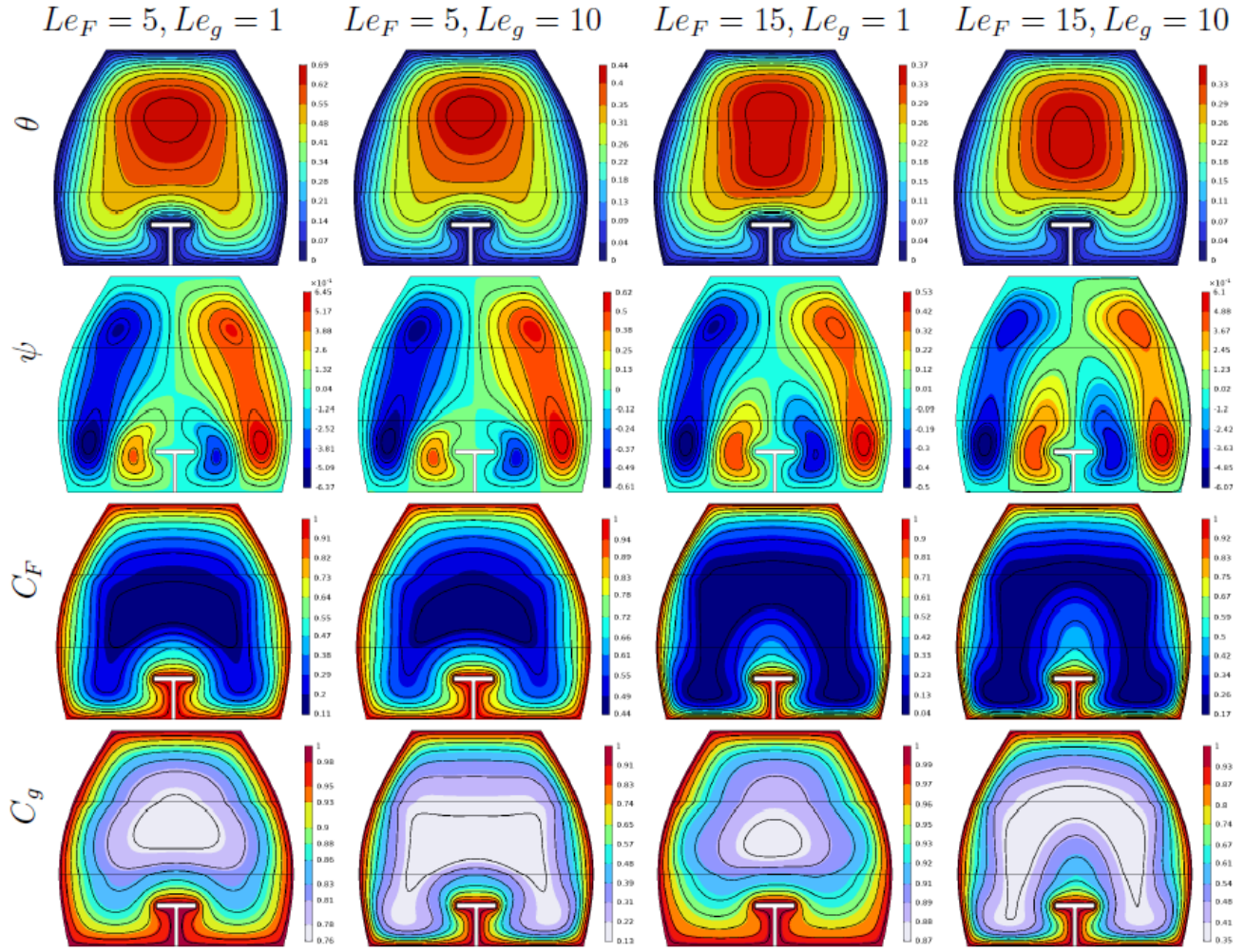


Figure 10: Contours of isotherms (θ), streamlines (ψ), fuel concentration (C_F), and gas concentration (C_g) for different Le_F and Le_g at $\phi = 0.03$, $Ra = 10^4$, $\delta = 15$.

Figure (10) examines the effects of the Lewis numbers of the fuel (Le_F) and the gas (Le_g). The results are obtained at $Ra = 10^4$, $\delta = 15$, and $\phi = 0.03$. The Lewis number represents the ratio of thermal diffusivity to mass diffusivity. Therefore, higher Lewis numbers indicate that species mass diffusion occurs much slower than thermal energy propagation. The top two rows show that the overall isotherm (θ) and streamline (ψ) patterns remain nearly unchanged, as the global flow structure is primarily dominated by the constant Rayleigh number. However, the maximum flow strength (ψ_{max}) and the peak temperature (θ_{max}) experience a slight reduction as the Lewis number increases. This indicates that the slower mass diffusion creates weaker concentration gradients, which in turn slightly reduces the assisting solutal buoyancy force. The most dramatic physical changes naturally happen in the species transfer fields. As the fuel Lewis number

increases from $Le_F = 5$ to 15, the fuel mass diffusivity drops significantly. The fuel concentration boundary layers (C_F) become thinner and steeper along the fin and solid walls. The central low concentration region expands and deepens. This reduces fuel diffusion into the bulk fluid of the partially porous layer cavity. The gas concentration (C_g) experiences a similarly restrictive effect. Raising the gas Lewis number from $1 \leq Le_g \leq 10$ pushes the concentration contours toward the boundaries, leaving a uniform central core. Ultimately, the reduced mass diffusivity traps the species near the active reaction zones instead of allowing them to mix thoroughly throughout the cavity.

Figure (11) examines the effects of the gas buoyancy parameter (N_g) and the fuel buoyancy parameter (N_F). The ranges considered are $0.5 \leq N_g \leq 10$ and $0.5 \leq N_F \leq 10$. The results are obtained at

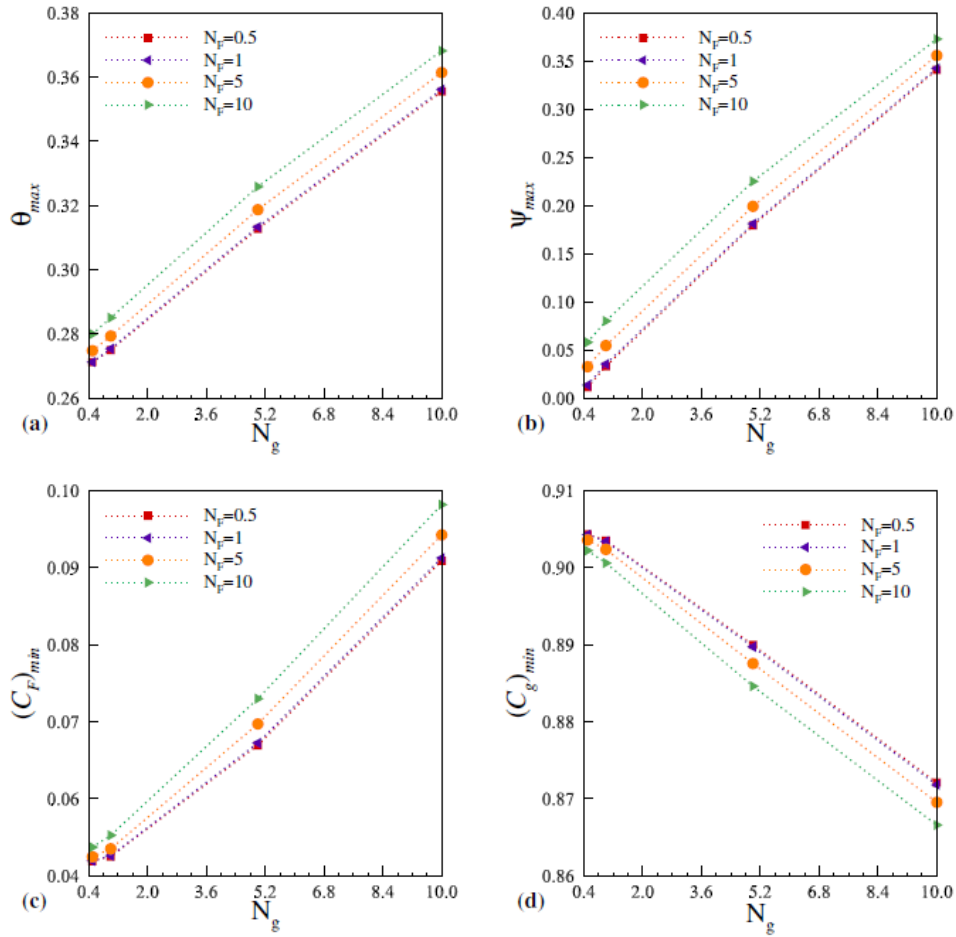


Figure 11: Influence of N_F and N_g on θ_{max} , ψ_{max} , $(C_F)_{min}$, and $(C_g)_{min}$ for $Ra = 10^3$, $\phi = 0.05$, $Le_F = 10$, $Le_g = 1$.

$Ra = 10^3$, with fixed values of $Le_F = 10$, $Le_g = 1$, and $\phi = 0.05$. Since base thermal convection remains weak at $Ra = 10^3$, concentration-driven buoyancy forces dictate the fluid motion. Increasing both N_g and N_F (from the red solid line to the green dashed line) produces a clear, nearly linear increase in the maximum flow intensity (ψ_{max}). This confirms that the solutal buoyancy from both species operates in tandem with the thermal buoyancy, providing a cumulative aiding effect that accelerates the convective flow. Stronger circulation improves heat transport. This supplies the exothermic reactions more effectively, pushing the peak temperature (θ_{max}) higher.

Although θ_{max} and ψ_{max} increase simultaneously, the species transfer reacts in a completely different way. Increasing the combined buoyancy forces (N_g and N_F) makes the flow stronger, which raises the minimum fuel concentration ($(C_F)_{min}$) by over 50%. This stronger mixing makes the fuel distribution more uniform, which prevents localized depletion. On the other hand, this strong buoyancy makes the minimum gas

concentration ($(C_g)_{min}$) drop quickly. This contrasting behavior is tied to their respective Lewis numbers. Because the fuel diffuses slowly ($Le_F = 10$), the enhanced advection helps spread it uniformly across the cavity. Conversely, the more diffusive gas ($Le_g = 1$) is rapidly fed directly into the active reaction zones and consumed, causing its local concentration to drop sharply.

Figure (12) tracks how global entropy generation (S_{global}), the Entropy Generation Ratio within the partially porous layer cavity (EGR), and the Bejan number (Be) shift across different Rayleigh numbers (Ra) and nanoparticle volume fractions (ϕ). This approach uses constant values ($Da = 10^{-3}$, $\delta = 5$) to track the exact effect of changing the porous layer thickness from (a) $L_p = 0.2$ to (b) $L_p = 0.4$.

Looking at the S_{global} curves, the total entropy generation climbs steadily as buoyancy forces strengthen with higher Ra . This upward trend accelerates past $Ra = 10^4$ because vigorous

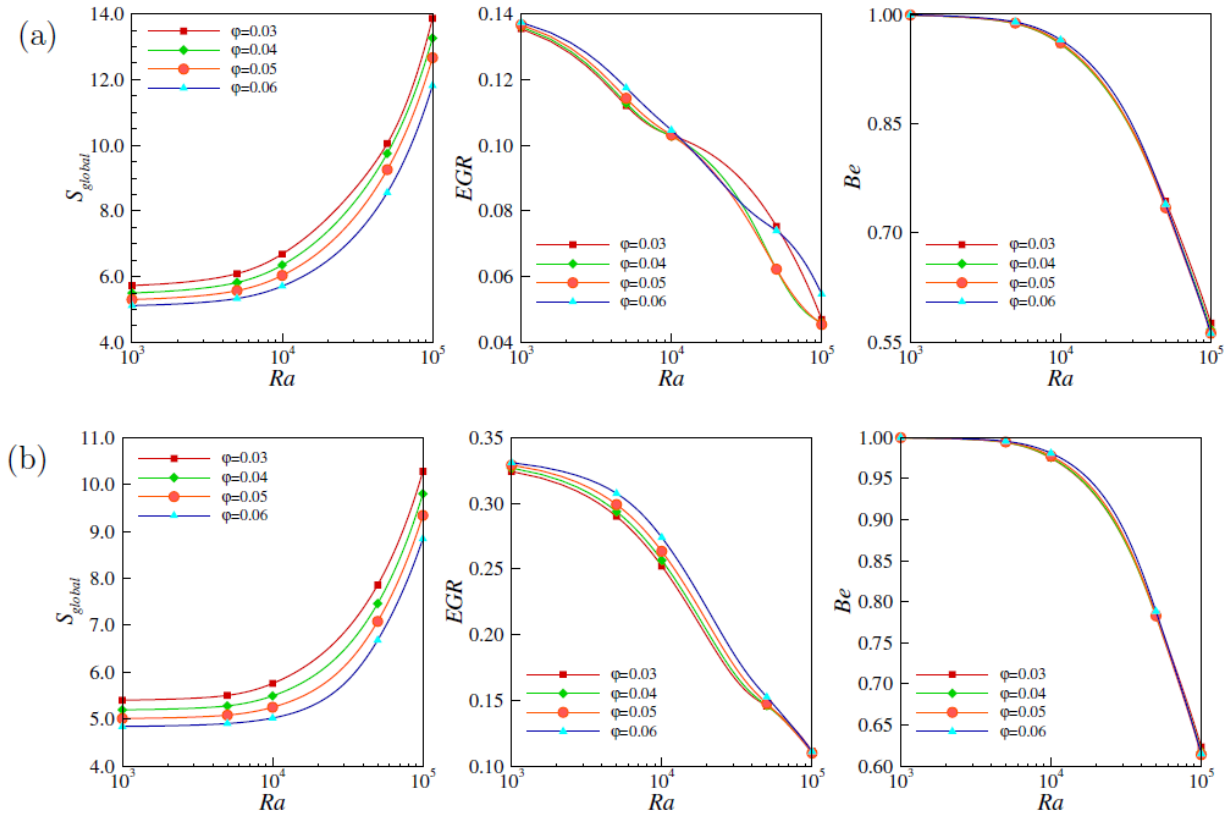


Figure 12: Variation of S_{total} , EGR , and Be for different Ra and ϕ at $Da = 10^{-3}$, $\delta = 5$, and (a) $L_p = 0.2$; (b) $L_p = 0.4$.

convection intensifies both thermal gradients and fluid friction across the enclosure. Increasing the nanoparticle volume fraction from $\phi = 0.03$ to 0.06 systematically reduces S_{global} . The higher nanoparticle concentration improves the effective thermal conductivity of the nanofluid. This reduces the temperature gradients and lowers the heat transfer irreversibility. Geometrically, stretching the porous layer from $L_p = 0.2$ to $L_p = 0.4$ clips the peak entropy at $Ra = 10^5$, dragging it down from roughly 13.5 to 10.0 a solid 26% reduction. Because the expanded porous layer slows fluid flow, it successfully reduces the entropy generated by fluid friction.

The EGR subplot reveals an inverse trend, peaking in the conduction heavy zone ($Ra = 10^3$) before sliding downward as convective forces take over at $Ra = 10^5$. Geometry strongly influences this behavior. Increasing the porous layer thickness from $L_p = 0.2$ to 0.4 shifts the entire EGR curve upward. The local contribution to entropy generation increases significantly. The maximum EGR reaches nearly 33% at $L_p = 0.4$. In contrast, it reaches only about 14% at $L_p = 0.2$. This corresponds to an increase of approximately 135% . Essentially, a thicker porous zone traps a larger fraction

of the total thermodynamic irreversibility within the porous region. The nanoparticle volume fraction (ϕ) has little influence on EGR at low Rayleigh numbers. A crossover occurs at $Ra = 10^5$, where the maximum nanoparticle concentration ($\phi = 0.06$) begins yielding slightly higher EGR values.

For the Bejan number (Be), the system holds steady near 100% as long as $Ra \leq 10^4$. When fluid motion remains weak, heat transfer resistance dictates almost all of the thermodynamic losses. Once Ra approaches 10^5 , however, Be decreases quickly to around 55% for $L_p = 0.2$ and 60% for $L_p = 0.4$. This sudden drop occurs because fast convective flows greatly increase fluid friction. Modifying ϕ yields nearly identical Be trajectories; adding nanoparticles alters the total entropy, but it does not shift the balance between thermal and frictional causes. The slightly higher Be percentage sustained at $Ra = 10^5$ for $L_p = 0.4$ proves that expanding the porous region restricts fluid momentum, keeping heat transfer as the slightly more dominant source of irreversibility.

Figure (13) shows the variations of S_{global} , EGR , and Be with the Frank-Kamenetskii number (δ) and the

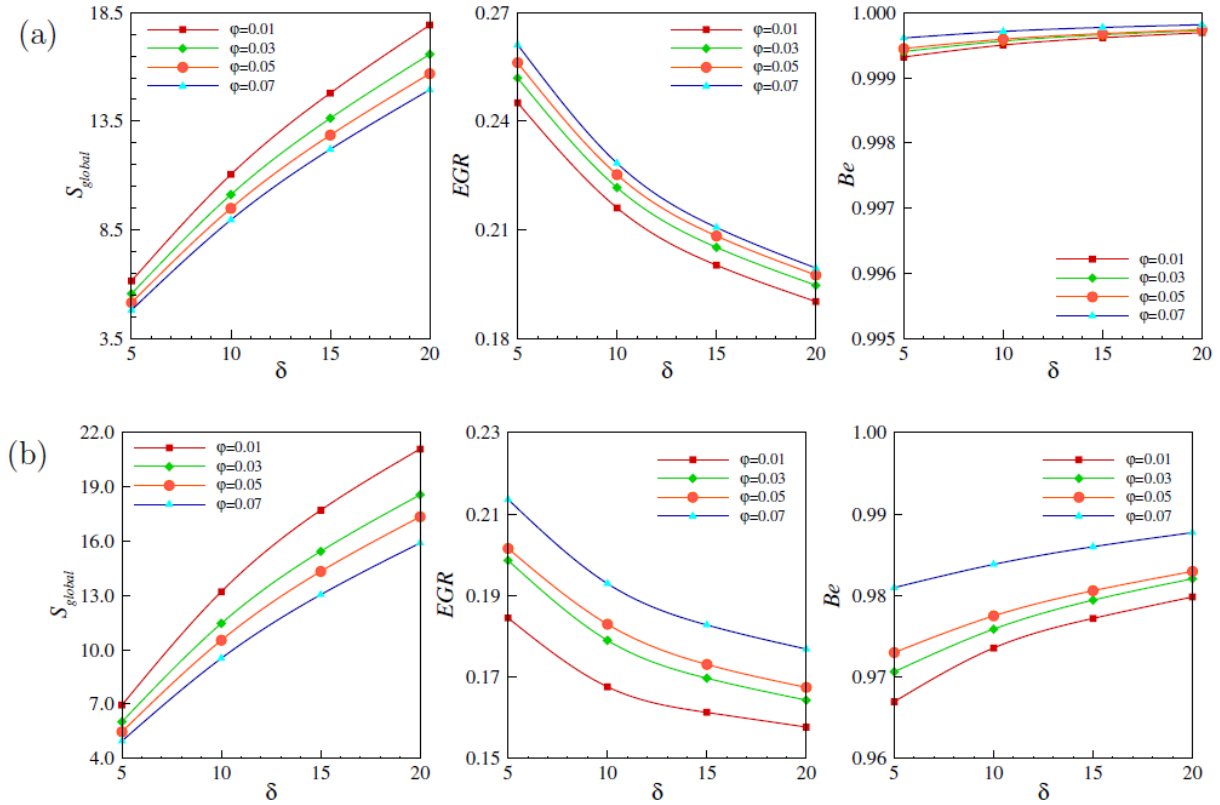


Figure 13: Variation of S_{total} , EGR, and Be for different δ and ϕ at $L_p = 0.33$, and (a) $Da = 10^{-2}$, $Ra = 10^3$; (b) $Da = 10^{-3}$, $Ra = 10^4$.

nanoparticle volume fraction (ϕ). The results are obtained at a fixed porous layer thickness of $L_p = 0.33$. The analysis examines two separate transport scenarios. (a) For highly permeable, low-buoyancy case ($Da = 10^{-2}$, $Ra = 10^3$) and (b) for more compact porous structure with stronger convection ($Da = 10^{-3}$, $Ra = 10^4$).

Looking at the S_{global} profiles, an upward adjustment in the Frank-Kamenetskii number from 5 to 20 strongly drives total thermodynamic irreversibility across both configurations. Under the stronger convective conditions of case (b), the global entropy generation increases significantly. The baseline value rises from approximately $4.0 \leq 7.0$ to peak values of about $15.5 \leq 21.0$. Increasing the Frank-Kamenetskii number (δ) enhances internal heat generation relative to thermal conduction. This accelerates thermodynamic irreversibility within the system. In contrast, increasing the nanoparticle volume fraction ($0.01 \leq \phi \leq 0.07$) suppresses this increase in entropy generation. At the highest δ mark in case (b), transitioning to the maximum nanoparticle loading pulls the absolute entropy down by roughly 26%. This drop occurs because the superior effective thermal conductivity of

the nanofluids, which smooths the internal temperature gradients and minimizes thermal energy losses.

The EGR subplot shows a contrasting trend, decreasing as the δ increases. In the high permeability environment of case (a), the porous domain claims roughly a 24.5% – 26.5% share of the total system losses at $\phi = 0.01$ and $\phi = 0.07$ when $\delta = 5$. However, this local contribution decreases with increasing δ , reaching about 18.5% – 20.0% at $\delta = 20$. Case (b) follows a similar pattern, where the contribution of the porous region to the total entropy generation decreases from approximately 21.3% to 15.5%. Adding nanoparticles shifts the thermodynamic burden back toward the porous layer; higher values of ϕ consistently lift the entire EGR curve. This upward shift indicates that the contribution of the partially porous layer to the total entropy generation increases. A higher nanoparticle concentration causes a larger fraction of the total irreversibility to be generated within the porous region.

The Bejan number (Be) dictates whether heat transfer or fluid friction dominates total entropy generation. For case (a), which operates under low

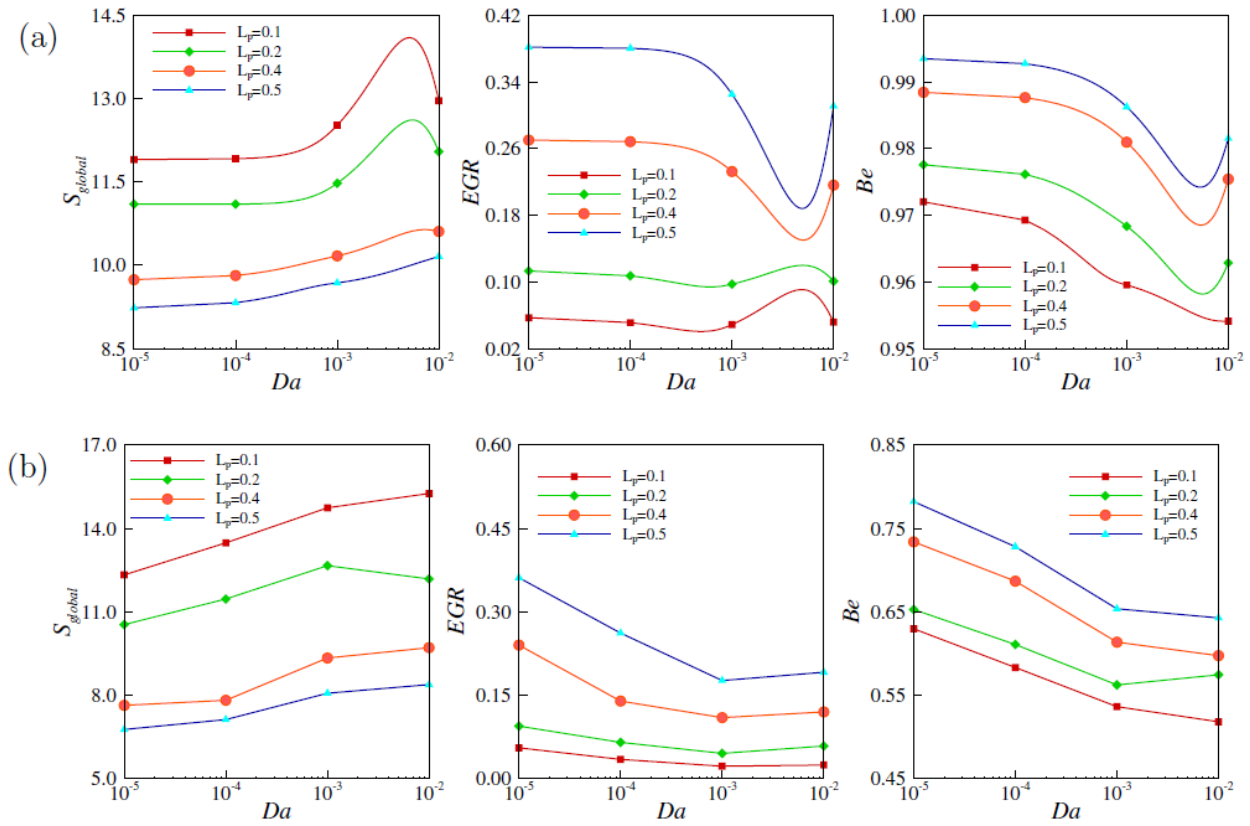


Figure 14: Variation of S_{total} , EGR, and Be for different Da and L_P at $\phi = 0.05$, and (a) $Ra = 10^4$, $\delta = 10$; (b) $Ra = 10^5$, $\delta = 5$.

convection, fluid friction plays almost no role; Be remains fixed near 99.9% to 100% across the whole δ spectrum, implying that fluid friction losses are practically nonexistent. However, case (b) presents a more dynamic balance due to the higher Ra . Here, the system starts with a slightly more active friction component at $\delta = 5$, yielding a Be value of 96.5% for $\phi = 0.01$. As δ climbs to 20, Be recovers steadily toward 98.0% for $\phi = 0.01$ and up to 98.8% for $\phi = 0.07$. Raising either δ or ϕ clearly dampens fluid motion. When this happens, heat transfer takes over as the primary cause of thermodynamic irreversibility within the partially porous layer cavity.

Figure (14) presents the effects of Da and L_P on S_{global} , EGR, and Be at $\phi = 0.05$, contrasting two flow scenarios: (a) $Ra = 10^4$, $\delta = 10$ and (b) $Ra = 10^5$, $\delta = 5$.

For the S_{global} profiles, higher Darcy numbers lead to increased entropy generation due to reduced resistance to fluid motion in a more permeable medium. In case (a), the profiles show a nonmonotonic trend for thinner porous layers ($L_P = 0.1$). This behavior appears in the high permeability range ($Da = 10^{-3}$ to 10^{-2}). Increasing the porous layer thickness from $L_P =$

0.1 to $L_P = 0.5$ effectively dampens the flow across all permeability ranges. For instance, in the highly convective regime of case (b) at $Da = 10^{-5}$, expanding the porous layer to $L_P = 0.5$ drops the absolute entropy generation from approximately 12.5 to 6.8, corresponding to a reduction of about 45.6%. This shows that a wider porous layer slows convective flow and lowers global thermodynamic losses.

EGR decreases with increasing Da , followed by a slight recovery at the highest permeability ($Da = 10^{-2}$). Porous layer thickness remains the dominant factor behind this effect. In case (a), at $Da = 10^{-5}$, a thin layer ($L_P = 0.1$) contributes approximately 5.5% to the system's total irreversibility. A thick layer ($L_P = 0.5$) contributes about 38.5%. This local contribution jumps by nearly 600%. A comparable trend holds true in case (b), where the maximum EGR drops from around 36% at $Da = 10^{-5}$ down to 20% at $Da = 10^{-3}$ for $L_P = 0.5$, showing how higher fluid permeability eventually shifts a larger fraction of thermodynamic losses outside the porous region.

The Bejan number (Be) outlines the shifting balance between thermal losses and fluid friction. Case (a)

represents a thermally dominated domain where Be remains above 95.5% across all Da . Thus fluid friction is almost negligible under these conditions. In contrast, case (b) shows a more dynamic behavior due to the higher $Ra = 10^5$, which induces stronger fluid motion and reduces Be significantly. As Da increases, Be drops steadily, indicating that fluid friction becomes increasingly important. However, thickening the porous layer from $L_p = 0.1$ to $L_p = 0.5$ consistently elevates the Be profiles. At $Da = 10^{-5}$ in case (b), this geometric modification elevates the thermal share from 63% up to 78%. This upward shift proves that a wider partially porous zone dampens fluid momentum, making heat transfer resistance the dominant cause of irreversibility.

7. CONCLUSIONS

Thermal energy transfer, fluid flow, and reactive species transport in a partially porous dome-shaped enclosure, incorporating the variable thermophysical properties of nanoparticles were analyzed numerically. This study systematically investigated the influences of stronger buoyancy driven convection, porous permeability, nanoparticle volume fraction, geometric configurations, and chemical reaction kinetics on transport processes and entropy generation. The main findings are as follows:

- At low Da , the porous matrix offers strong resistance to the flow. This reduces flow intensity and shifts the transport behavior from convection dominated to conduction dominated. As a result, the active species region shifts upward in the cavity. Both ψ_{max} and θ_{max} decrease by about 43% and 30%, respectively.
- Higher Ra enhances fluid mixing and strengthens convection. However, increasing φ yields competing effects. Heat transfer is improved, and the peak temperature (θ_{max}) is reduced by about 19%. At the same time, the effective viscosity increases, which slightly weakens the flow and reduces ψ_{max} by around 7%. At the same time, it helps maintain higher minimum species concentrations, with $(C_F)_{min}$ and $(C_g)_{min}$ increasing by up to 12%.
- Physical obstacles alter the fluid flow by changing its circulation paths. A fin on the bottom wall disturbs the flow and pushes heat and species upward, while a fin on the top wall has the opposite effect, driving them downward. In the same way, increasing the porous layer thickness (L_p) acts like a barrier that blocks the upper part of

the cavity and confines most of the active mixing to the lower clear region.

- For exothermic chemical transport, higher Le_F and Le_g thin the species boundary layers. This confines transport near the walls and weakens mixing, which leads to a reduction in both flow strength and temperature. In contrast, increasing δ strengthens heat generation, raising θ_{max} by up to 46% and increasing ψ_{max} by about 34%, while sharply reducing $(C_F)_{min}$ and $(C_g)_{min}$ by roughly 32% and 78%, respectively. However, increasing both N_g and N_F increases $(C_F)_{min}$ but decreases $(C_g)_{min}$.
- S_{global} increases with stronger convection effects (Ra , δ) and higher permeability (Da), but decreases with nanoparticle loading (φ) and porous thickness (L_p), with reductions reaching about 26%–45.6% depending on the case.
- The EGR is mainly governed by the porous structure, increasing sharply with L_p (up to nearly 600% in local contribution), while generally decreasing with higher Da and δ , and showing only mild sensitivity to φ .
- The Be number remains close to unity (95% : 100%) in weak flow regimes, but drops significantly at high Ra to about 55%–60%, while increasing with both L_p and δ , indicating a gradual shift toward heat transfer dominated irreversibility.

DECLARATION OF INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

AVAILABILITY OF DATA

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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AUTHORS CREDIT STATEMENT

NCR: Conceptualization; Formal analysis; Investigation; Methodology; Supervision; Writing – original draft; Writing – review & editing; Validation.

MJT: Conceptualization; Data curation; Investigation; Methodology; Validation; Writing – original draft; Writing – review & editing; Visualization.

RSRG: Writing – original draft; Writing – review & editing; Investigation; Data curation; Methodology.

NOMENCLATURE

Symbols

A	Pre-exponential factor ($\text{m}^3\text{kg}^{-1}\text{s}^{-1}$)
Be	Bejan number
c_F	Concentration of fuel (molL^{-1})
c_g	Concentration of oxidizer (molL^{-1})
c_{F0}	Initial concentration of fuel (molL^{-1})
c_{g0}	Initial concentration of gas (molL^{-1})
C_F	Dimensionless concentration of fuel
C_g	Dimensionless concentration of gas
C_p	Specific heat capacity ($\text{Jkg}^{-1}\text{K}^{-1}$)
d	Equivalent diameter of molecule (nm)
d_p	Pore diameter (mm)
Da	Darcy number
D_F	Diffusivity of nanofluid fuel (m^2s^{-1})
D_g	Diffusivity of oxidizer (m^2s^{-1})
E	Activation Energy (Jmol^{-1})
EGR	Entropy generation ratio
F_0	Forchheimer coefficient
g	Gravitational acceleration (ms^{-2})
H_R	Heat of reaction parameter
k	Thermal conductivity ($\text{Wm}^{-1}\text{K}^{-1}$)
K	Permeability (m^2)
Le_F	Lewis number for fuel
Le_g	Lewis number for gas
N_F	Buoyancy ratio parameter for fuel
N_g	Buoyancy ratio parameter for gas
p	Dimensional pressure (Pa)
P	Dimensionless pressure
Pr	Prandtl number
Q	Heat of reaction (Jkg^{-1})
R	Universal gas constant ($\text{Jmol}^{-1}\text{K}^{-1}$)
Ra	Rayleigh number
S_θ	Entropy generation due to thermal gradients

S_F	Entropy generation due to viscous dissipation
S_D	Entropy generation due to concentration gradients
S_{por}	Dimensionless entropy generation in porous layer
S_{gen}	Entropy generation rate ($\text{WK}^{-1}\text{m}^{-3}$)
S_{total}	Dimensionless entropy generation rate
S_{global}	Dimensionless global entropy generation
T	Dimensional temperature (K)
T_0	Environment temperature (K)
$\tilde{T}$	Reference temperature ($^{\circ}\text{C}$)
u, v	Dimensional velocity components (ms^{-1})
U, V	Dimensionless velocity components
x, y	Cartesian coordinates (m)
X, Y	Dimensionless Cartesian coordinates

Greek Symbols

α	Thermal diffusivity (m^2s^{-1})
δ	Frank-Kamenetskii number
β	Coefficient of thermal expansion (K^{-1})
$\beta_{c,F}$	Solute expansion coefficient for fuel (Lmol^{-1})
$\beta_{c,g}$	Solute expansion coefficient for gas (Lmol^{-1})
γ	Zel'dovich number
κ	Stoichiometric ratio
Π	Irreversibility distribution parameter
ϵ	Porosity
θ	Dimensionless temperature
μ	Dynamic viscosity ($\text{kgs}^{-1}\text{m}^{-1}$)
ν	Kinematic viscosity (m^2s^{-1})
ρ	Density (kgm^{-3})
ϕ	Volume fraction of Nanoparticle
Φ	Volume fraction (Percentage)
ψ	Stream-function

Subscripts

bf	Base fluid
f	Quantities in fluid layer
nf	Nanofluid
np	Nano particle
p	Quantities in porous layer
s	Solid matrix

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