Concentration Gradient Driven Mixed Convective Flow in Rectangular Enclosure with Concentrated Top Wall

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Abstract: Concentration gradient driven mixed convective flow in a horizontal rectangular enclosure with concentrated top wall is studied in this work. The continuity, momentum and mass transfer equations are discretized by using the hybrid finite volume method. The innovative thought in this work is to examine the sole effect of the concentration gradient on the mass transfer between the top wall and the fluid within the horizontal rectangular enclosure. The dimensionless parameters such as the Reynolds number (*Re*), the Schmidt number (*Sc*) are appropriately chosen to calculate the Richardson numbers (*Ri*) for specific fluids such as ammonia gas (Sc = 0.78) and methanol liquid (Sc = 556) at low (Re = 25) and high (*Re* = 25000) Reynolds numbers at different mass transfer Rayleigh numbers ($Ra_{m,L}$) in the range of $3\times10^5 \leq Ra_{m,L}) \leq 7\times10^9$. In accordance with these calculated Richardson numbers revealed as *Ri*>>1 and *Ri*<<1 respectively, it is found that pure and negligible free convection effects prevails between the top wall and the fluid within the horizontal rectangle. When *Ri*~1 mixed convection results between the top wall to these fluids within the horizontal rectangle. A novel empirical correlation for calculating the average Sherwood numbers ($\overline{Sh}$) for specific fluids at different mass transfer Rayleigh numbers as mentioned above is suggested in this research and thereby the mass transfer from the top wall to these fluids within the horizontal rectangle is examined and validated. Since this correlation is novel in nature, the conclusions regarding the mass transfer from the top wall to these fluids within the horizontal rectangle justified and comparison or improvement over the existing correlations is unnecessary. It is found that for all the boundary values of the mass transfer Rayleigh number, there is a substantial increase in the average Sherwood numbers $\overline{Sh}$ and hence the mass transfer from the top wall to the fluid (ammonia gas or methanol liquid) substantially increases within the horizontal rectangle.

Keywords: Horizontal rectangular enclosure, hybrid finite volume method, empirical correlation, Reynolds number, Rayleigh number, mass transfer, average Sherwood number.

1. INTRODUCTION

Mass transfer driven flow is a topic of great significance and has numerous industrial applications in chemical and aerospace industry. There are several applications which involves the mass transfer such as absorption, solvent extraction, evaporation of water vapour into the dry air, diffusion of smoke from a chimney into the atmosphere, evaporation of petrol in internal combustion engines.

Hydromagnetic double-diffusive convection in a rectangular enclosure with opposing temperature and concentration gradients was investigated by Chamkha and Al-Naser [2]. Laminar double-diffusive mixed convection in two-dimensional displacement ventilated enclosure with discrete heat and contaminated sources was numerically studied by Deng *et al*. [3]. Momentum and heat transport for conjugate free convective flow by using streamline and heatline was numerically visualized by Deng and Tang [4]. Globe and Dropkin [5] have proposed an empirical correlation for average Nusselt number for different Rayleigh numbers of a natural convective heat transfer in liquids confined by two horizontal plates heated from below. Hollands *et al*. [6] have proposed empirical correlations for average Nusselt number of free convective air and water flow in horizontal rectangular region. Patankar and Spalding [7] have introduced a benchmark method for calculating the flow variables linked to a three-dimensional mass, momentum and energy equations. Takhar *et al*. [8] have presented the solutions both analytically and numerically for thermal and mass diffusion along moving vertical slender cylinder. Ambethkar and Basumatary [10, 11] have investigated the numerical solutions of steady free convective flow in rectangular region with discrete wall heat and concentration sources.

An inducement to the current research is because of an inadequate research on mass driven natural convective flow in a rectangular enclosures. Particularly during the past three decades, the mass transfer driven flow in a horizontal rectangular enclosure with a concentrated top wall has not been extensively studied by the researchers. Therefore, in order to set right this defect, the present research is carried out.

The innovation in the present research is to examine the sole effect of the concentration gradient

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and associated mass transfer due to the top wall on convective flow within the horizontal rectangle. An empirical correlation for calculating the average Sherwood numbers ($\overline{Sh}$) for the above mentioned specific fluids such as ammonia gas ($Sc = 0.78$) and methanol liquid ($Sc = 556$) is suggested and thereby the mass transfer between the wall and these fluids for different mass transfer Rayleigh numbers is examined. Since this correlation is novel in nature, the conclusions regarding the mass transfer from the top wall to these fluids within the horizontal rectangle justified and comparison or improvement over the existing correlations is unnecessary. It is found that for all the boundary values of the mass transfer Rayleigh number, there is a substantial increase in the average Sherwood numbers $\overline{Sh}$ and hence the mass transfer from the top wall to the fluid (ammonia gas or methanol liquid) substantially increases within the horizontal rectangle.

The objective of this study is to discretize the continuity, momentum and mass transfer equations by using the hybrid finite volume scheme. The dimensionless parameters such as the Reynolds number (Re), the Schmidt number (Sc) are appropriately chosen so as to calculate the Richardson numbers (Ri) for specific fluids such as ammonia gas ($Sc = 0.78$) and methanol liquid ($Sc = 556$) at low ($Re = 25$) and high ($Re = 25000$) Reynolds numbers and at different Rayleigh numbers. Furthermore, in accordance with these calculated Richardson numbers revealed as $Ri \ll 1$ and $Ri \gg 1$ respectively, it is found that pure and negligible free convection effects prevails between the top wall and the fluid within the horizontal rectangle. When $Ri \approx 1$ mixed convection results between the top wall to these fluids within the horizontal rectangle.

2. PROBLEM FORMULATION

2.1. Schematic Information

A horizontal rectangular enclosure $ABCD$ that is of length H and height L having a contaminant source of high concentration on top wall is displayed in Figure

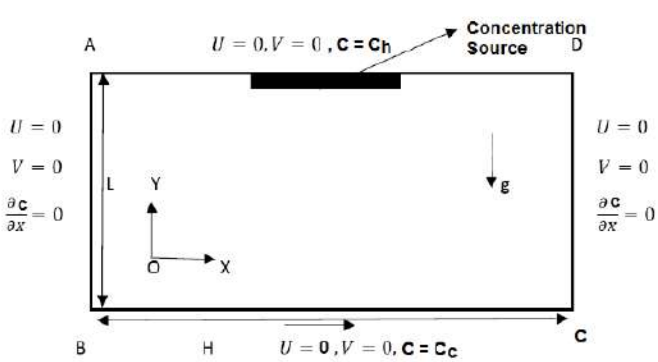


Figure 1: Schematic of the problem.

1. On all the four walls of the rectangular enclosure, no-slip boundary conditions are defined for velocity. In order to sustain the mass driven convective flow within $ABCD$, uniform concentration flux type boundary conditions on the top and bottom walls are defined. The uniform concentration flux boundary conditions are applicable at all points except the four corner points A, B, C, D of the rectangular enclosure.

2.2. Governing Equations

The mass transfer driven free convective flow in a horizontal rectangle as displayed in Figure **1** is governed by the continuity, the momentum and the mass transfer equation written in dimensional form as follows [1]:

$$\text{3continuityequation:} \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{1}$$

$$\text{X}-\text{momentumequation:} U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{1}{\rho}\frac{\partial p}{\partial X} + \nu\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right), \tag{2}$$

$$\text{Y}-\text{momentumequation:}$$
$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{1}{\rho}\frac{\partial p}{\partial Y} + \nu\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + g\beta_C(C - C_c), \tag{3}$$

$$\text{masstransferequation:} U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y} = D\left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2}\right). \tag{4}$$

where U, V, X, Y, p, C, ρ, ν, g, β_C, C_c and D are the dimensional velocity components along X and Y-directions, independent variables, pressure, concentration, density, kinematic viscosity, acceleration due to gravity, coefficient of thermal expansion at constant concentration, concentration of the cold wall and mass diffusivity respectively.

The boundary conditions corresponding to equations (1)–(4) in dimensional form are defined as:

$$\left.\begin{array}{lll} \text{onAB:} & \text{at } X = 0,\ U = 0, V = 0,\ \dfrac{\partial C}{\partial X} = 0, \\ \text{onDC:} & \text{at } X = H,\ U = 0, V = 0,\ \dfrac{\partial C}{\partial X} = 0, \\ \text{onBC:} & \text{at } Y = 0,\ U = 0, V = 0,\ C = C_c, \\ \text{onAD:} & \text{at } Y = L,\ U = 0, V = 0,\ C = C_h. \end{array}\right\} \tag{5}$$

Where H and L are the length and height of the horizontal rectangle.

We introduce the dimensionless variables as follows [10]:

$$6(x, y) = \frac{(X, Y)}{L}, \quad (u, v) = \frac{(U, V)}{u_0}, \quad P = \frac{p}{\rho u_0^2}, c = \frac{C - C_c}{C_h - C_c}$$

$$Gr_c = \frac{g\beta_C(C_h - C_c)}{\nu^2}L^3, \quad Re = \frac{Lu_0}{\nu}, \quad Sc = \frac{\nu}{D}, \quad Ri = \frac{Gr_c}{Re^2}.$$

where x, y, L, u, v, u_0, P, c, Gr_c, Re, Sc and Ri are dimensionless independent variables along X and Y directions, height of the rectangle, dimensionless velocity components along x and y-directions, characteristic velocity, dimensionless pressure, dimensionless concentration, the Grashof number with reference to the concentration, the Reynolds number, the Schmidt number and the Richardson number. The subscripts h, and c, represents the hot and cold conditions.

The governing equations (1)–(4) in dimensionless form takes the following form [1]:

$$\text{6continuityequation:}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{6}$$

$$\text{x – momentumequation:}u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{1}{Re}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{7}$$

y – momentumequation:

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + cRi, \tag{8}$$

$$\text{masstransferequation: } u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} = \frac{1}{ReSc}\left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2}\right). \tag{9}$$

The boundary conditions in dimensionless form reduces to:

$$\left.\begin{array}{lll} \text{onAB:} & \text{at } x = 0, & u = 0, v = 0, \; \dfrac{\partial c}{\partial x} = 0, \\ \text{onDC:} & \text{at } x = 2, & u = 0, v = 0, \; \dfrac{\partial c}{\partial x} = 0, \\ \text{onBC:} & \text{at } y = 0, & u = 0, v = 0, \; c = 0, \\ \text{onAD:} & \text{at } y = 1, & u = 0, v = 0, \; c = 1. \end{array}\right\} \tag{10}$$

where AB, DC, BC, and AD are the left, right, bottom and top walls of the rectangle.

3. DISCRETIZATION

3.1. Discretization of the Governing Equations

The governing equations of the present problem comprises of the continuity, momentum and mass transfer equations. The main difficulty in finding a solution to the governing equations is because of their nonlinearity and pressure-velocity coupling. Therefore, it is difficult to compute a solution in usual manner and thus, an iterative solution strategy ought to be utilized as a method of solution. The idea of discretization is to use an unusual staggered gird in which the scalar flow variables P and c are defined at the nodes and the velocity variables are defined at the faces of the control volumes. Hence, the governing equations are discretized by means of the hybrid finite volume scheme [9, p.195-200~] and a modified Semi-Implicit Method for Pressure-Linked equation (SIMPLE) algorithm is utilized for obtaining accurate numerical solutions of the flow variables.

Continuity equation: The discretized continuity equation at location (I, J) is given by

$$F_e - F_w + F_n - F_s = 0 \tag{11}$$

x-momentum: The discretized u-momentum equation at location (i, J) is given by

$$a_{i,J}u_{i,J} = \sum a_{nb}u_{nb} + (P_{I-1,J} - P_{I,J})A_{i,J} \tag{12}$$

where $A_{i,J}$ is the cell face areas of u-control volume. E, W, N, S neighbors involved in the summation $\sum a_{nb}u_{nb}$ are ($i+1, J$), ($i-1, J$), ($i, J+1$) and ($i, J-1$) in accordance with [9, p.197-199~]. The coefficients of hybrid differencing scheme are

$$a_{i,J} = a_{i-1,J} + a_{i+1,J} + a_{i,J+1} + a_{i,J-1} + \Delta F \tag{13}$$

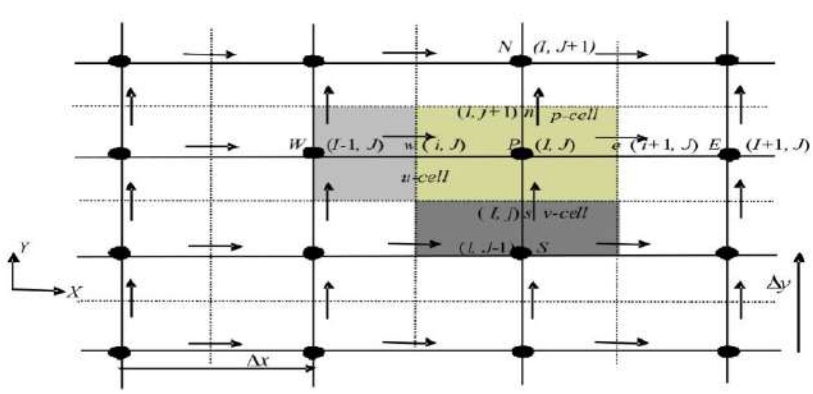


Figure 2: Staggered grid arrangement.

where

$$\left.\begin{array}{l} a_{i+1,J} = max[-F_{I,J},(D_{I,J} - \frac{F_{I,J}}{2}),0], a_{i-1,J} = max[F_{I-1,J},(D_{I-1,J} + \frac{F_{I-1,J}}{2}),0], \\ a_{i,J+1} = max[-F_{i,j+1},(D_{i,j+1} - \frac{F_{i,j+1}}{2}),0], a_{i,J-1} = max[F_{i,j},(D_{i,j} + \frac{F_{i,j}}{2}),0], \\ \Delta F = (F_e - F_w) + (F_n - F_s) = (F_{I,J} - F_{I-1,J}) + (F_{i,j+1} - F_{i,j}) \end{array}\right\}$$

For each of the faces e, w, n and s, we have

$$\left.\begin{array}{l} F_e = F_{I,J} = \dfrac{F_{i+1,J} + F_{i,J}}{2} = \dfrac{u_{i+1,J}A_{i+1,J} + u_{i,J}A_{i,J}}{2} \\ F_w = F_{I-1,J} = \dfrac{F_{i,J} + F_{i-1,J}}{2} = \dfrac{u_{i,J}A_{i,J} + u_{i-1,J}A_{i-1,J}}{2} \\ F_n = F_{i,j+1} = \dfrac{F_{I,j+1} + F_{I-1,j+1}}{2} = \dfrac{v_{I,j+1}A_{I,j+1} + v_{I-1,j+1}A_{I-1,j+1}}{2} \\ F_s = F_{i,j} = \dfrac{F_{I,j} + F_{I-1,j}}{2} = \dfrac{v_{I,j}A_{I,j} + v_{I-1,j}A_{I-1,j}}{2} \\ D_e = D_{I,J} = \dfrac{A_{I,J}}{Re\Delta x} = \dfrac{1}{Re\Delta x}(\dfrac{A_{i+1,J} + A_{i,J}}{2}) \\ D_w = D_{I-1,J} = \dfrac{A_{I-1,J}}{Re\Delta x} = \dfrac{1}{Re\Delta x}(\dfrac{A_{i,J} + A_{i-1,J}}{2}) \\ D_n = D_{i,j+1} = \dfrac{A_{i,j+1}}{Re\Delta y} = \dfrac{1}{Re\Delta y}(\dfrac{A_{I,j+1} + A_{I-1,j+1}}{2}) \\ D_s = D_{i,j} = \dfrac{A_{i,j}}{Re\Delta y} = \dfrac{1}{Re\Delta y}(\dfrac{A_{I,j} + A_{I-1,j}}{2}) \end{array}\right\} \quad (14)$$

y-momentum: The discretized v-momentum equation at (I, j) is given by

$$a_{I,j}v_{I,j} = \sum a_{nb}v_{nb} + (P_{I,J-1} - P_{I,J})A_{I,j} + b_{I,j} \quad (15)$$

where $A_{i,J}$ is the cell face areas of v-control volume. The neighbours E, W, N and S involved in the summation $\sum a_{nb}v_{nb}$ are ($I+1, j$), ($I-1, j$), ($I, j+1$) and ($I, j-1$)in accordance with [9, p.199-200~].

Now the coefficients of upwind differencing scheme are as follows:

$$a_{I,j} = a_{I+1,j} + a_{I-1,j} + a_{I,j+1} + a_{I,j-1} + \Delta F - S_{I,j} \quad (16)$$

and

$$S_{I,j}v_{I,j} + b_{I,j} = \overline{S}\Delta v \quad (17)$$

where

$$\left.\begin{array}{c} a_{I+1,j} = max[-F_{i+1,j},(D_{i+1,j} - \frac{F_{i+1,j}}{2}),0], a_{I-1,j} = max[F_{i,j},(D_{i,j} + \frac{F_{i,j}}{2}),0], \\ a_{I,j+1} = max[-F_{I,J},(D_{I,J} - \frac{F_{I,J}}{2}),0], a_{I,j-1} = max[F_{I,J-1},(D_{I,J-1} + \frac{F_{I,J-1}}{2}),0], \\ \Delta F = (F_e - F_w) + (F_n - F_s) = (F_{i+1,j} - F_{i,j}) + (F_{I,J} - F_{I,J-1}) \end{array}\right\}$$

As per the notations used in v-control volume, the values of F and D, for each of the faces e, w, n and s reduce to

$$\left.\begin{array}{l} F_e = F_{i+1,j} = \dfrac{F_{i+1,J} + F_{i+1,J-1}}{2} = \dfrac{u_{i+1,J}A_{i+1,J} + u_{i+1,J-1}A_{i+1,J-1}}{2} \\ F_w = F_{i,j} = \dfrac{F_{i,J} + F_{i,J-1}}{2} = \dfrac{u_{i,J}A_{i,J} + u_{i,J-1}A_{i,J-1}}{2} \\ F_n = F_{I,J} = \dfrac{F_{I,j} + F_{I,j+1}}{2} = \dfrac{v_{I,j}A_{I,j} + v_{I,j+1}A_{I,j+1}}{2} \\ F_s = F_{I,J-1} = \dfrac{F_{I,j-1} + F_{I,j}}{2} = \dfrac{v_{I,j-1}A_{I,j-1} + v_{I,j}A_{I,j}}{2} \\ D_e = D_{i+1,j} = \dfrac{A_{i+1,J}}{Re\Delta x} = \dfrac{1}{Re\Delta x}(\dfrac{A_{i+1,J} + A_{i+1,J-1}}{2}), D_w = D_{i,j} = \dfrac{A_{i,j}}{Re\Delta x} = \dfrac{1}{Re\Delta x}(\dfrac{A_{i,J} + A_{i,J-1}}{2}) \\ D_n = D_{I,J} = \dfrac{A_{I,J}}{Re\Delta y} = \dfrac{1}{Re\Delta y}(\dfrac{A_{I,j} + A_{I,j+1}}{2}), D_s = D_{I,J-1} = \dfrac{A_{I,J-1}}{Re\Delta y} = \dfrac{1}{Re\Delta y}(\dfrac{A_{I,j-1} + A_{I,j}}{2}) \end{array}\right\} \quad (18)$$

The pressure correction equation as per [9, p.202~] reduces to

$$a_{I,J}P'_{I,J} = a_{I+1,J}P'_{I+1,J} + a_{I-1,J}P'_{I-1,J} + a_{I,J+1}P'_{I,J+1} + a_{I,J-1}P'_{I,J-1} + b'_{I,J} \quad (19)$$

where

$$a_{I,J} = a_{I+1,J} + a_{I-1,J} + a_{I,J+1} + a_{I,J-1}$$

and the coefficients are

$$\left.\begin{array}{l} a_{I+1,J} = (dA)_{i+1,J},\ a_{I-1,J} = (dA)_{i,J}, \\ a_{I,J+1} = (dA)_{I,j+1},\ a_{I,J-1} = (dA)_{I,j-1}, \\ d_{i,J} = \dfrac{A_{i,J}}{a_{i,J}},\ d_{I,j} = \dfrac{A_{I,j}}{a_{I,j}} \\ b'_{I,J} = (u^*A)_{i,J} - (u^*A)_{i+1,J} + (v^*A)_{I,j} - (v^*A)_{I,j+1} \end{array}\right\} \quad (20)$$

The equation (19) represents the discretized continuity equation as an equation of pressure correction P'. The source term b' in the equation is the continuity imbalance equation arising from the incorrect velocity field u^* and v^*. By solving equation (19), the pressure correction field P' could be obtained at all points.

Mass transfer equation: The discretized mass transfer equation at (I,J) is given by

$$a_{I,J}C_{I,J} = \sum a_{nb}C_{nb} \quad (21)$$

The coefficients of the hybrid differencing scheme are

$$a_{I,J} = a_{I+1,J} + a_{I-1,J} + a_{I,J+1} + a_{I,J-1} + \Delta F \quad (22)$$

where

$$\left.\begin{array}{c} a_{I+1,J} = max[-F_{i+1,J},(D_{i+1,J} - \frac{F_{i+1,J}}{2}),0], a_{I-1,J} = max[F_{i,J},(D_{i,J} + \frac{F_{i,J}}{2}),0], \\ a_{I,J+1} = max[-F_{I,j+1},(D_{I,j+1} - \frac{F_{I,j+1}}{2}),0], a_{I,J-1} = max[F_{I,j},(D_{I,j} + \frac{F_{I,j}}{2}),0], \\ \Delta F = (F_e - F_w) + (F_n - F_s) = (F_{i+1,J} - F_{i,J}) + (F_{I,j+1} - F_{I,j}) \end{array}\right\}$$

and the values of F and D for each of the faces e, w, n and s can be expressed in a similar manner as given above.

3.2. Numerical Solutions of the Flow Variables

Numerical Solutions of the flow variables u, v, P, c are calculated from the discretized equations of the u-momentum, the v-momentum, the pressure correction equation and the mass transfer equations by appropriately choosing the parameters $Ra_{m,L}$, Re and Sc. We compose an algorithm that enable us to the solve these discretized equations iteratively. Algorithm is summarized as follows:

Algorithm

The notorious Semi-Implicit Method for Pressure-Linked equation (SIMPLE) algorithm See [9, p.204~] is appropriately modified and thereby the numerical solutions of pressure, velocities and concentration are calculated in a iterative manner. Main steps involved in this algorithm are summarized as:

Step 1: Start with guessed velocities (u^*, v^*) pressure fields P^* and concentration c^*.

Step 2: Calculate the coefficients in the momentum equation, solve discretized momentum equations.

Step 3: Calculate the coefficients of pressure equation, solve pressure correction equations.

Step 4: Correct pressure and velocities:

$$\left.\begin{aligned} P_{I,J} &= P^*_{I,J} + P'_{I,J}, \\ u_{i,J} &= u^*_{i,J} + d_{i,J}(P'_{I-1,J} - P'_{I,J}), \\ v_{I,j} &= v^*_{I,j} + d_{I,j}(P'_{I,J-1} - P'_{I,J}). \end{aligned}\right\} \tag{23}$$

Step 5: Solve concentration discretized equations.

Step 6: Replace the previous intermediate values of pressure, velocity and concentration (P^*, u^*, v^*, c^*) with the corrected values (P, u, v, c), return to Step 2 and repeat this process until the solution converges.

4. ANALYSIS OF RESULTS

In the present research, our aim is to analyze the nature of the mass transfer driven mixed convective flow within the rectangular enclosure due to a concentrated top wall. Therefore, the Richardson numbers (Ri) for specific fluids such as ammonia gas ($Sc = 0.78$) and methanol liquid ($Sc = 556$) at low ($Re = 25$) and high ($Re = 25000$) Reynolds numbers (Re) are calculated for different mass transfer Rayleigh numbers ($Ra_{m,L}$) in the range of $3\times10^5 \le Ra_{m,L} \le 7\times10^9$, where $Ra_{m,L}$ is defined as $Ra_{m,L} = Gr_c Sc$ and Gr_c is the mass Grashof number which is defined in section 2.2. It is to be noted that the Schmidt numbers for these two fluids as mentioned above are valid at low concentration of 1 atm and $25^\circ C$ (approx.). The Richardson numbers (Ri) for a specific fluid such as ammonia gas ($Sc = 0.78$) at low ($Re = 25$) and high ($Re = 25000$) Reynolds numbers (Re) for different mass transfer Rayleigh numbers are calculated and sketched in Figure **3**. It is observed from Figures **3a** and **3b** that, At low and high Reynolds numbers ($Re = 25$ and $Re = 25000$), as the mass transfer Rayleigh numbers for ammonia gas ($Sc = 0.78$) increases, there is an exponential and slight increase in the Richardson number respectively. However, for all the boundary values of the mass transfer Rayleigh number, there is a substantial increase ($Ri \gg 1$) in the Richardson numbers and hence, it can be concluded that pure natural convection effects prevails between the top wall and the fluid. The reason for this behaviour is that pure free convection prevails if the calculated Richardson numbers (Ri) satisfy $Ri \gg 1$ and a negligible free convection prevails if $Ri = 1$ within the given range of the mass transfer Rayleigh numbers ($Ra_{m,L}$) as mentioned above. When $Ri \approx 1$ mixed convection results between the top wall to these fluids within the horizontal rectangle.

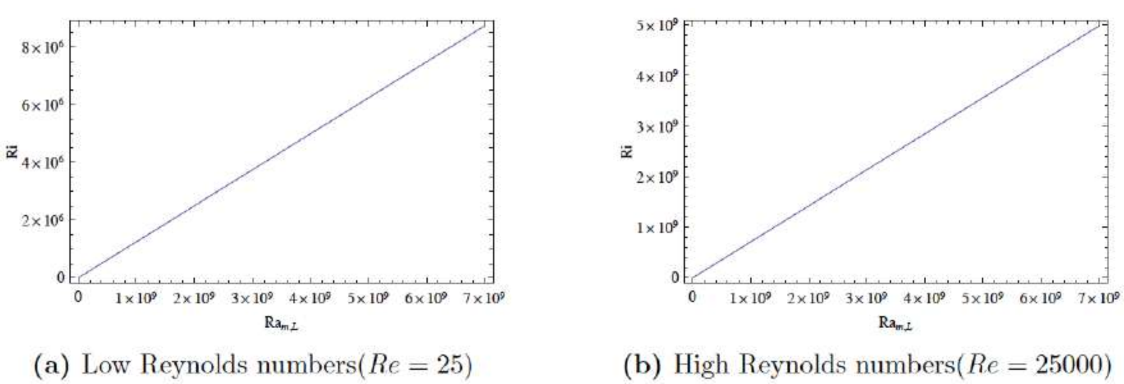
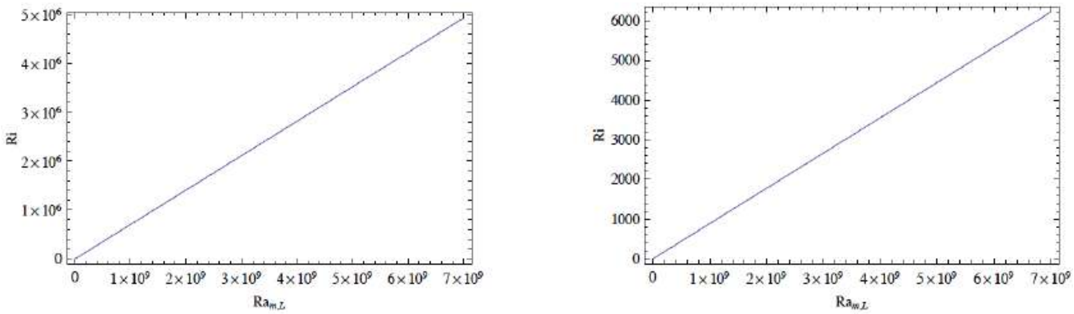


Figure 3: Richardson number (Ri) for ammonia gas at different mass transfer Rayleigh numbers ($Ra_{m,L}$).



(a) Low Reynolds numbers($Re = 25$) (b) High Reynolds numbers($Re = 25000$)

Figure 4: Richardson number (Ri) for methanol liquid at different mass transfer Rayleigh numbers ($Ra_{m,L}$).

Analogously, the Richardson numbers (Ri) for methanol liquid ($Sc = 556$) at low ($Re = 25$) and high ($Re = 25000$) Reynolds numbers (Re) are calculated for different mass transfer Rayleigh numbers ($Ra_{m,L}$) in the range of $3\times10^5 \le Ra_{m,L} \le 7\times10^9$. We have appropriately chosen the Schmidt number for methanol liquid as $Sc = 556$ at low concentration of 1 atm and $25^{\circ}C$ (approx.). The Richardson numbers (Ri) for a specific fluid such as the methanol liquid ($Sc = 556$) at low ($Re = 25$) and high ($Re = 25000$) Reynolds numbers (Re) are calculated and sketched in Figure **4**. It is observed from Figures **4a** and **4b** that, At low and high Reynolds numbers ($Re = 25$ and $Re = 25000$), as the mass transfer Rayleigh number for methanol liquid ($Sc = 556$) increases, there is an exponential and slight increase in the Richardson number respectively. However, for all the boundary values of the mass transfer Rayleigh number, there is a slight increase ($Ri = 1$) in the Richardson numbers and hence negligible free convection effects prevails between the top wall and the fluid in the horizontal rectangle and hence negligible mixed convection effects prevails between the top wall and the fluid in the horizontal rectangle. However, for all the boundary values of the mass transfer Rayleigh number, there is a substantial increase ($Ri \gg 1$) in the Richardson numbers and hence, it can be concluded that pure natural convection effects prevails between the top wall and the fluid in the horizontal rectangle. The reason for this behaviour is that pure free convection prevails if the calculated Richardson numbers (Ri) satisfy $Ri \gg 1$ and a negligible free convection prevails if $Ri = 1$ within the given range of the mass transfer Rayleigh numbers ($Ra_{L,m}$) as mentioned above. When $Ri \approx 1$ mixed convection results between the top wall to these fluids within the horizontal rectangle.

5. VALIDATION OF THE MASS TRANSFER RESULTS

Since the goal of this present study is to investigate the mass transfer between the top wall and fluid within the horizontal rectangle, an empirical correlation for the average Sherwood number $\overline{Sh} = 0.069(Ra_{m,L})^{\frac{1}{3}} Sc^{0.074}$; $3\times10^5 \le Ra_{m,L} \le 7\times10^9$ is proposed in this investigation which does not exist in the literature. The average Sherwood numbers ($\overline{Sh}$) for specific fluid such as gas ammonia ($Sc = 0.781$) and a liquid methanol ($Sc = 556$) at different mass transfer Rayleigh numbers in the range of $3\times10^5 \le Ra_{m,L} \le 7\times10^9$ are calculated by using this empirical correlation and tabulated in Tables **1** and **2**. It is observed from Table **1** that for ammonia gas ($Sc = 0.781$), as the mass transfer Rayleigh number increases, there is a slight increase in the average Sherwood number $\overline{Sh}$ and hence the mass transfer increases between the top wall and fluid within the horizontal rectangle. However, for all the boundary values of the mass transfer Rayleigh number, there is a substantial increase in the average Sherwood number $\overline{Sh}$ and hence the mass transfer increases substantially between the top wall and fluid (ammonia gas) within the horizontal rectangle.

Analogously, It is noticed from Table **2** that for methanol liquid ($Sc = 556$), as the mass transfer

Table 1: Average Sherwood Number of a Gas Ammonia $(Sc = 0.781)$ at Different Mass Transfer Rayleigh Numbers $(Ra_{m,L})$

Type of fluid(Gas)	Mass transfer Rayleigh number $(Ra_{m,L})$	Average Sherwood numbers
Ammonia (Sc=0.781)	3×10^5	4.53537
	3×10^9	97.7115
	4×10^5	4.99182
	5×10^5	5.37727
	6×10^5	5.7142
	7×10^5	6.01549
	7×10^9	129.6

Rayleigh number increases, there is a slight increase and hence the mass transfer increases between the top wall and fluid within the horizontal rectangle. However, for all the boundary values of the mass transfer Rayleigh number, there is a substantial increase in the average Sherwood number $\overline{Sh}$ and hence the mass transfer increases substantially between the top wall and fluid (methanol liquid) within the horizontal rectangle. Hence, it is evident from these calculations of the average Sherwood numbers $\overline{Sh}$ which are described in the following Tables **1** and **2** that the conclusions drawn for the mass transfer between the top wall and fluids within the horizontal rectangle are thus validated. When $Ri \approx 1$ mixed convection results between the top wall to these fluids within the horizontal rectangle.

Table 2: Average Sherwood Number of a Liquid Methanol $(Sc = 556)$ at Different Mass Transfer Rayleigh Numbers $(Ra_{m,L})$

Type of fluid(Liquid)	Mass transfer Rayleigh number $(Ra_{m,L})$	Average Sherwood numbers
Methanol (Sc=556)	3×10^5	7.37381
	3×10^9	158.864
	4×10^5	8.11592
	5×10^5	8.74261
	6×10^5	9.29041
	7×10^5	9.78027
	7×10^9	210.709

6. CONCLUSION

- At low and high Reynolds numbers ($Re = 25$ and $Re = 25000$), as the mass transfer Rayleigh numbers for ammonia gas ($Sc = 0.78$) increases, there is an exponential and slight increase in the Richardson number respectively.

- At low and high Reynolds number ($Re = 25$ and $Re = 25000$), as the mass transfer Rayleigh number for methanol liquid ($Sc = 556$) increases, there is an exponential and slight increase in the Richardson number respectively.

- An empirical correlation for the average Sherwood number $\overline{Sh} = 0.069(Ra_{m,L})^{\frac{1}{3}} Sc^{0.074}$; $3\times10^5 \le Ra_{m,L} \le 7\times10^9$ is proposed in this investigation.

- The average Sherwood numbers ($\overline{Sh}$) for specific fluid such as gas ammonia ($Sc = 0.781$) and a liquid methanol ($Sc = 556$) at different mass transfer Rayleigh numbers are calculated by using this empirical correlation.

- For ammonia gas ($Sc = 0.781$), as the mass transfer Rayleigh number increases, there is a slight increase in the average Sherwood number $\overline{Sh}$ and hence the mass transfer from the top wall to this fluid slightly increases within the horizontal rectangle.

- For all the boundary values of the mass transfer Rayleigh number, the mass transfer from the top wall to the fluid for ammonia gas substantially increases within the horizontal rectangle.

- The mass transfer from the top wall to the fluid for methanol liquid ($Sc = 556$) substantially increases within the horizontal rectangle.

- The mass transfer between the top wall and the fluid within the horizontal rectangle is validated from the calculations of the average Sherwood numbers $\overline{Sh}$.

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